C-Conserving Decay $\eta \to \pi^0 e^+ e^-$ in a Vector-Meson-Dominant Model

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(Received 25 May 1967)

The decay $\eta \to \pi e^+ e^-$ through a C-conserving, two-photon intermediate state is studied in a vector-meson-dominant model. The branching ratio $\Gamma(\eta \to \pi e^+ e^-)/\Gamma(\eta \to \pi\gamma\gamma)$ is found to be $\approx 10^{-5}$. If, in addition, a quark model is assumed for the photon–pseudoscalar–vector-meson vertex, we obtain a value of 1.3×10^{-5} eV for the decay width of $\eta \to \pi e^+ e^-$. General features of the decay spectrum with respect to the sum and difference of the electron and positron energies in the η rest frame are discussed.

I. INTRODUCTION

It has been pointed out by Bernstein, Feinberg, and Lee[1] that the decay $\eta \to \pi^0 e^+ e^-$ through a single-photon intermediate state can occur if the electromagnetic interactions violate charge-conjugation invariance. They estimated that, unless the C-violating current has special isospin or unitary-spin transformation properties, the rate of this decay should be appreciable if the violation is large. However, recent experimental results[2] have put an upper limit of less than 10^{-3} for the ratio of $\eta \to \pi^0 e^+ e^-$ decay width to the total η width. Anticipating a very small decay rate, we examine here the decay mode of $\eta \to \pi e^+ e^-$ through a C-conserving, two-photon intermediate state. Knowledge of this process will be important in the proper interpretation of future experimental data. Hereafter in this paper, the decay $\eta \to \pi^0 e^+ e^-$ means the C-conserving one, unless otherwise stated.

We first discuss some general features of the decay amplitude M. To be Lorentz invariant, M should be of the form

$$M = \sum_i C_i \psi_e \Gamma_i \psi_e ,$$

where i goes over the five convariants, namely, S, V, T, A, and P, and where the C_i's are functions of external momenta. Because of the behavior of the Dirac equation, M should be invariant under the combined transformations,

$$\psi_e \to \gamma_5 \psi_e \quad \text{and} \quad m_e \to -m_e .$$

As it is well known that S, T, and P change sign under the γ_5 transformation, their corresponding C_i's must then be proportional to some odd powers of electron mass. We shall neglect all electron-mass terms as small.[3] By parity conservation, the decay amplitude must then be of the following form:

$$M = G\bar{\psi}_e \gamma_\lambda \psi_e P_\lambda + H\bar{\psi}_e \gamma_\lambda \gamma_5 \psi_e \epsilon_{\lambda\mu\nu\rho} P_\mu P_{+\nu} P_{-\rho} , \qquad (1)$$

where P, P_+, and P_- are, respectively, the 4-momenta of η, e^+, and e^-. $\epsilon_{\lambda\mu\nu\rho}$ is the totally antisymmetric unit tensor of rank 4. The form factors G and H depend in general on two independent invariants of the external momenta, for example, $P(P_+ - P_-)$ and $P(P_+ + P_-)$. In the rest frame of η, we take the two independent variables to be $S = \omega_+ + \omega_-$ and $D = \omega_+ - \omega_-$, where ω_+ and ω_- are the positron and electron energy variables. In order for M to be evan under charge conjugation, it is necessary that G and H be odd functions of D, since the covariants multiplying G and H are both odd under C. Clearly, amplitudes of such a form will give a decay spectrum that is symmetric with respect to ω_+ and ω_-, and will vanish for $\omega_+ = \omega_-$. We expect the axial-vector term to be less important because of its strong momentum dependence.

All the features so far discussed are general; independent of the models assumed, the decay amplitude must be of the form shown in Eq. (1) in the limit of electron mass $m_e = 0$. We shall now proceed in Sec. II to examine in detail the decay $\eta \to \pi^0 e^+ e^-$ in a vector-meson-dominant (VMD) model.[4] In Sec. III a quark model is used to evaluate the coupling constants involved in the photon–pseudoscalar–vector-meson vertex. We find the decay width $\Gamma(\eta \to \pi e^+ e^-) = 1.3 \times 10^{-5}$ eV. In Sec. IV the ratio $\Gamma(\eta \to \pi e^+ e^-)/\Gamma(\eta \to \pi\gamma\gamma)$ is computed to be $\approx 10^{-5}$. In Sec. V, the last result is checked in a different model in which the photons emerge from the same vertex and the γ-γ-η-π vertex is assumed to take the general form $P_\alpha P_\beta F_{\alpha\mu} F_{\beta\mu} \phi_\eta \phi_\pi$. Here $F_{\alpha\beta}$ is the electromagnetic field tensor.

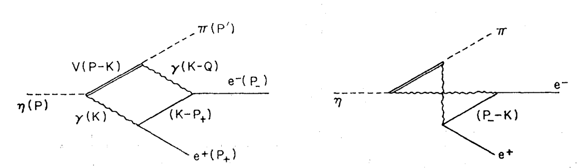

FIG. 1. Feynman diagrams for $\eta \to \pi\gamma\gamma \to \pi e^+ e^-$ in a VMD model.

II. VECTOR-MESON-DOMINANT MODEL

We assume in the following calculation that the decay $\eta \to \pi^0 e^+ e^-$ is dominated by the virtual transitions $\eta \to V^0 \gamma$ followed by $V^0 \to \pi^0 \gamma$ and $2\gamma \to e^+ e^-$. The

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[1] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[2] See, for example, C. Baglin, A. Bezaguet, H. H. Bingham, B. Degrange, F. Jacquet, W. Michael, P. Musset, U. Nguyen-Khai, and G. Nihoul-Boutang, Phys. Letters **22**, 219 (1966).

[3] We do not expect any mass singularity $\sim 1/m_e$ to occur. For the problem of mass singularity in the Feynman diagrams see, for example, T. Kinoshita, J. Math. Phys. **3**, 650 (1962).

[4] See, for example, M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

Feynman diagrams of this process are shown in Fig. 1. V^0 denotes all known neutral vector mesons of zero strangeness, namely, ω, ρ^0, and ϕ. Thus, there are in fact a total of six diagrams. We take the simplest pseudoscalar (P)-photon (A_μ)-vector (V_μ) vertex consistent with the requirements of Lorentz, C, P, and gauge invariance,

$$g_{PV\gamma}(\epsilon_{\alpha\beta\gamma\sigma}\partial_\alpha V_\beta\partial_\gamma A_\sigma)P\,, \tag{2}$$

where $g_{PV\gamma}$ is some appropriate coupling constant.

The invariant decay amplitude (M) in momentum space is then

$$\begin{aligned}M=&\sum_{V=\omega,\rho,\phi} g_V(M_V^{(1)}-M_V^{(2)})\\ =&\sum_V g_Ve^2\int\frac{d^4K}{(2\pi)}\frac{1}{K^2+i\epsilon}\frac{1}{(K-Q)^2+i\epsilon}\left[\epsilon_{\alpha\beta\gamma\sigma}(K-Q)_\alpha(P-K)_\gamma\frac{\delta_{\sigma\rho}-(P-K)_\sigma(P-K)_\rho m_V^{-2}}{(P-K)^2-m_V^2+i\epsilon}\epsilon_{\lambda\mu\nu\rho}K_\lambda(P-K)_\nu\right]\\ &\times\bar{U}(P_-,S_-)\left[\gamma_\beta\frac{1}{(K-P_+)\gamma-m_e+i\epsilon}\gamma_\mu+\gamma_\mu\frac{1}{(P_--K)\gamma-m_e+i\epsilon}\gamma_\beta\right]U(-P_+,S_+)\,,\end{aligned} \tag{3}$$

where $g_V=g_{\eta V\gamma}g_{\pi V\gamma}$ and $Q=P_++P_-$. $M_V^{(1)}$ and $M_V^{(2)}$ are amplitudes corresponding to uncrossed and crossed photon-line diagrams in Fig. 1. Because of the presence of the totally antisymmetric tensors $\epsilon_{\alpha\beta\gamma\sigma}$ and $\epsilon_{\lambda\mu\nu\rho}$ the term proportional to $(P-K)_\sigma(P-K)_\rho m_V^{-2}$ in the vector-meson propagator vanishes and, in fact, each vertex can contribute at most one power of K to the numerator. As a result the integral over d^4K is convergent as $K\to\infty$. In the following calculation we shall always set the electron mass $m_e=0$ and also neglect, in the numerator, terms that are second order in P_+ and P_-.

After rationalizing the electron propagator, we obtain from the numerator of $M_V^{(1)}$ a term of the following type:

$$\begin{aligned}N=&\epsilon_{\alpha\beta\gamma\sigma}(Q-K)_\alpha(P-K)_\gamma\epsilon_{\lambda\mu\nu\sigma}K_\lambda(P-K)_\nu\gamma_\beta[(K-P_+)\cdot\gamma]\gamma_\mu\\ =&\epsilon_{\alpha\beta\gamma\sigma}\epsilon_{\lambda\mu\nu\sigma}[Q_\alpha P_\gamma P_\nu\gamma_\beta\gamma_\rho\gamma_\mu K_\lambda K_\rho-P_\gamma P_\nu\gamma_\beta\gamma_\rho\gamma_\mu K_\alpha K_\lambda K_\rho-Q_\alpha P_\nu\gamma_\beta\gamma_\rho\gamma_\mu K_\gamma K_\lambda K_\rho+P_\gamma P_\nu\gamma_\beta(\gamma\cdot P_+)\gamma_\mu K_\alpha K_\lambda].\end{aligned} \tag{4}$$

By means of Feynman parameters,

$$\begin{aligned}M_V^{(1)}=&e^2\int\frac{d^4K}{(2\pi)^4}\frac{\bar{U}(P_-,S_-)NU(-P_+,S_+)}{(K^2+i\epsilon)[(K-Q)^2+i\epsilon][(P-K)^2-m_V^2+i\epsilon][(K-P_+)^2+i\epsilon]}\\ =&e^2 3!\int\frac{d^4K}{(2\pi)^4}\int_0^1dz\int_0^{1-z}dy\int_0^{1-z-y}dx\frac{\bar{U}(P_-,S_-)NU(-P_+,S_+)}{(K^2-2b_\lambda K_\lambda+a_V+i\epsilon)^4}\,,\end{aligned} \tag{5}$$

where

$$a_V=(m_\eta^2-m_V^2)z+2(P_+\cdot P_-)y\,,$$
$$b_\lambda=P_{+\lambda}x+Q_\lambda y+P_\lambda z\,.$$

We can then perform the d^4K integration by change of variables and contour rotation. Thus, for example

$$\int d^4K\frac{K_\lambda K_\mu K_\nu}{(K^2-2bK+a_V)^4}=\frac{i\pi^2}{12c_V}[b_\lambda\delta_{\mu\nu}+b_\mu\delta_{\lambda\nu}+b_\nu\delta_{\lambda\mu}+(2/c_V)b_\lambda b_\mu b_\nu]\,,$$

where $c_V=a_V-b_\lambda b_\lambda$. After neglecting second-order terms of P_+ and P_- in the numerator and after a considerable amount of algebra, we obtain

$$M_V^{(1)}=\bar{U}(\gamma P)U\frac{ie^2}{8\pi^2}\int dxdydz\frac{P(P_--xP_+-yQ+1.5zP)}{c_V+i\epsilon}. \tag{6}$$

$M_V^{(2)}$ is of similar form to $M_V^{(1)}$ with P_+ and P_- interchanged. (We note that the denominator c_V is also a function of P_+ and P_-). Thus our decay amplitude is in complete agreement with the vector form we have expected from very general arguments in Sec. I [see Eq. (1)]. We note parenthetically that, had we used a model with photons emerging from the same vertex (see Fig. 2) and assumed for the γ-γ-π-η vertex the simplest possible interaction $\phi_\eta\phi_\pi F_{\mu\nu}F_{\mu\nu}$ ($F_{\mu\nu}$ is the electromagnetic field tensor), we would not be able to obtain an amplitude of the form found in Eq. (1), because this interaction in fact gives a decay amplitude proportional to the electron mass. Thus, we expect that contributions from such an interaction will be several orders of magnitude smaller than those from the present VMD model.[5]

[5] C. H. Llewellyn Smith [Nuovo Cimento 48, 834 (1967)] has used just such a model of pure S-wave coupling, $F_{\mu\nu}F_{\mu\nu}\phi_\pi\phi_\eta$. The ratio $\Gamma(\eta\to\pi e^+e^-)/\Gamma(\eta\to\pi\gamma\gamma)$ is found to be $\approx10^{-8}$ with a reasonable cutoff for the divergent integral. Similar results have been obtained by T. J. Weare [Imperial College (London) Report 67-13 (unpublished)] and J. Smith [University of Adelaide Report (unpublished)].

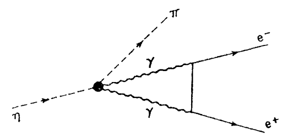

FIG. 2. Feynman diagrams for $\eta \rightarrow \pi\gamma\gamma \rightarrow \pi e^+e^-$ with two photons emerging from the same vertex.

III. NUMERICAL EVALUATION

To obtain an absolute rate for the decay, we need to to know the value of g_V [see Eqs. (2) and (3)]. For this purpose we shall assume a quark model, which has been rather successful in predicting, among other things, the rate of $\omega \rightarrow \pi^0\gamma$.[6] In the quark model[7] the pseudoscalar mesons (P) are the ${}^1S_0$ state of a quark-antiquark system and vector mesons (V) are the ${}^3S_1$ states. A $V \leftrightarrow P+\gamma$ process is therefore interpretated as an $M1$ transition of quarks. The amplitude is then associated with the magnetic moments of quarks, which are in turn related to the total proton magnetic moment $\mu_p = 2.79e/2m_p$. When ω-ϕ and X-η mixings are taken into account,[8] the quark model predicts[9]

$$g_\omega = 2.2\mu_p{}^2, \quad g_\rho = 1.0\mu_p{}^2, \quad g_\phi = 0.$$

Furthermore, because of the near equality of ω and ρ masses, it is a good approximation to equate the amplitudes $M_\omega{}^{(1)}$ ($M_\omega{}^{(2)}$) and $M_\rho{}^{(1)}$ ($M_\rho{}^{(2)}$), their only difference being an $m_V{}^2$ term in the denominator c_V. Consequently the g_V's can be factored out in the expression for decay width, $\Gamma \sim |\sum_V g_V(M_V{}^{(1)} - M_V{}^{(2)})|^2$.

We consider the decay spectrum with respect to S and D, the sum and difference of electron and positron energies. As anticipated, $d\Gamma/dD$ is a symmetric function with respect to positive and negative values of D, and vanishes at $D=0$. S is linearly related to $Q^2 = (P_+ + P_-)^2$. In our approximation of setting $m_e = 0$, Q^2 varies from zero to $(m_\eta - m_\pi)^2 \approx 17\times 10^4$ MeV^2, corresponding to 0° and 180° angles between the emerging electron-positron pair.

The real and imaginary parts of the amplitude in Eq. (6) are separated by the familiar identity,

$$\frac{1}{c_V + i\epsilon} = \text{P.V.}\frac{1}{c_V} - i\pi\delta(c_V),$$

where P.V. means the principal value. Contributions by the real and imaginary amplitudes to decay width will be denoted by subscripts R and I, respectively.

[5] T. J. Weare [Imperial College (London) Report 67–13 (unpublished)] has used just such a model with a $F_{\mu\nu}F_{\mu\nu}\phi_\pi\phi_\eta$ vertex. The ratio $\Gamma(\eta \rightarrow \pi e^+e^-)/\Gamma(\eta \rightarrow \pi\gamma\gamma)$ is found to be $\approx 10^{-7}$ with a cutoff for the divergent integral set at the nucleon mass.

[6] W. Thirring, Phys. Letters **16**, 335 (1965); V. V. Anisovitch, A. A. Anselm, Ya. I. Azimov, G. S. Danilov, and I. T. Dyatlov, *ibid.* **16**, 194 (1965).

[7] See, for example, R. H. Dalitz, *The Quark Model for Elementary Particles* (Gordon and Breach Science Publishers, Inc., New York, 1965).

[8] We took mixing angles to be $\cos\theta_{\omega\phi} = \sqrt{\frac{2}{3}}$ and $\tan\theta_{X\eta} = -0.2$.

[9] Reference 7, Table 3, p. 298. (The table contains some misprints.)

TABLE I. Differential decay spectrum of $\eta \rightarrow \pi^0e^+e^-$.

Q^2 [(100 MeV)^2]	S (MeV)	$d\Gamma_R/dS$	$d\Gamma_I/dS$
1.0	266	9.5×10^{-14}	2.1×10^{-13}
2.3	279	5.0×10^{-14}	1.5×10^{-13}
3.7	291	3.2×10^{-14}	1.1×10^{-13}
5.0	303	2.2×10^{-14}	7.5×10^{-14}
6.4	315	1.5×10^{-14}	5.0×10^{-14}
7.7	328	9.5×10^{-15}	3.2×10^{-14}
9.1	340	5.8×10^{-15}	1.9×10^{-14}
10.4	352	3.2×10^{-15}	1.1×10^{-14}
11.7	364	1.6×10^{-15}	5.5×10^{-15}
13.0	377	7.1×10^{-16}	2.4×10^{-15}
14.4	389	2.3×10^{-16}	7.5×10^{-16}
15.7	401	3.7×10^{-17}	1.2×10^{-16}
17.0	412	0	0

The integrations are done numerically; the results for differential width are shown in Table I and are plotted in Fig. 3.

The differential decay width corresponding to the dispersive part of the process, $d\Gamma_R/dS$, displays a $(\ln Q^2)^2$ dependence. Indeed for small Q^2 the spectrum can be very well fitted by $[\ln(Q^2/21)]^2\times10^{-14}$. Using this function to extrapolate the spectrum to $Q^2=0$, we are able to compute the width, $\Gamma_R = 0.4\times10^{-5}$ eV. The differential width of the absorptive process, $d\Gamma_I/dS$, increases less rapidly with Q^2 and is expected to be regular at $Q^2=0$. We obtain $\Gamma_I = 0.9\times10^{-5}$ eV, thus giving the total decay width

$$\Gamma(\eta \rightarrow \pi e^+e^-) = 1.3\times10^{-5}\ \text{eV}. \tag{7}$$

IV. $\Gamma(\eta \rightarrow \pi^0e^+e^-)/\Gamma(\eta \rightarrow \pi^0\gamma\gamma)$ RATIO (VMD MODEL)

Admittedly, the procedure of obtaining the g_V's through a quark model is not very reliable. For one thing there are ambiquities in the identification of the V-P-γ coupling constants within the model.[10] Nevertheless we can arrive at an experimentally measurable result that is more or less model-independent.

We note that $\eta \rightarrow \pi^0\gamma\gamma$ involves the same set of g_V's. Thus we expect the branching ratio $\Gamma(\pi^0e^+e^-)/\Gamma(\pi^0\gamma\gamma)$ will be independent of the values assumed for the coupling constants which can be, as we have seen, factored out in the expression for decay width. We compute the rate for $\eta \rightarrow \pi^0\gamma\gamma$ in the same VMD model.

The invariant decay amplitude is

$$M = \sum_V g_V \epsilon_{\alpha\beta\gamma\sigma}\epsilon_{\lambda\mu\nu\sigma} q_\alpha (p-k)_\gamma k_\lambda (p-k)_\nu \times\left(\frac{\epsilon_\beta\epsilon_\mu{}'}{(p-k)^2 - m_V{}^2 + i\epsilon} + \frac{\epsilon_\mu\epsilon_\beta{}'}{(p-q)^2 - m_V{}^2 + i\epsilon}\right), \tag{8}$$

where p, k, and q are the 4-momenta for η and two photons, which have polarizations ϵ and ϵ', respectively. To obtain the decay width, we square the amplitude and sum over all final states. A factor of $\frac{1}{2}$ will be inserted to account for the identical particles in the

[10] Reference 7, p. 296.

final state. The integrations are again done numerically, giving $\Gamma(\eta \to \pi^0\gamma\gamma)=1.3$ eV.[11]

Thus we have for the branching ratios,

$$\frac{\Gamma_R(\eta \to \pi^0 e^+ e^-)}{\Gamma(\eta \to \pi^0\gamma\gamma)} \approx 0.3\times 10^{-5}, \tag{9}$$

$$\frac{\Gamma_I(\eta \to \pi^0 e^+ e^-)}{\Gamma(\eta \to \pi^0\gamma\gamma)} \approx 0.7\times 10^{-5}, \tag{10}$$

and it follows that

$$\frac{\Gamma(\eta \to \pi^0 e^+ e^-)}{\Gamma(\eta \to \pi^0\gamma\gamma)} \approx 10^{-5}. \tag{11}$$

We note that this final result is computed in a VMD model with the electron mass set equal to zero throughout. We have also dropped, in the numerator of the amplitude, terms that are second order in P_+ and P_-. Among them, the most dangerous term is $Q^2=(P_+ + P_-)^2$. Since the final result is of such form that the principal contributions come from small Q^2, we expect this approximation to be a good one, or, properly speaking, a self-consistent one.

V. $\Gamma_I(\eta \to \pi e^+ e^-)/\Gamma(\eta \to \pi\gamma\gamma)$ IN A FORM-FACTOR MODEL

As a check for the results obtained in the VMD model we compute $\Gamma_I(\eta \to \pi e^+ e^-)/\Gamma(\eta \to \pi\gamma\gamma)$, the ratio of the absorptive process $\eta \to \pi e^+ e^-$ to the width of of $\eta \to \pi\gamma\gamma$, in a different model.[12] We assume for the η-π-γ-γ vertex a coupling of $P_\alpha P_\beta F_{\alpha\mu} F_{\beta\mu} \phi_\pi \phi_\eta$, where P_λ is the η 4-momentum. We note that this form-factor-type coupling satisfies all the invariance requirements states in the Introduction and it gives a nonzero amplitude in the limit of $m_e=0$.[13] Explicitly, the invariant absorptive amplitude is (the kinematics are the same as those used in the VMD model)

$$\begin{aligned} M_I(\pi e^+ e^-) = {} & g e^2 \bar{U}(P_-,S_-) \int \frac{d^4K}{(2\pi)^4} i\pi\delta(K^2) i\pi\delta((K-Q)^2) \\ & \times [(KP)\delta_{\mu\alpha} - K_\mu P_\alpha]\{[(Q-K)\cdot P]\delta_{\mu\beta} - (Q-K)_\mu P_\beta\} \\ & \times \left[\gamma_\alpha \frac{1}{\gamma\cdot(K-P_+)-m_e}\gamma_\beta + \gamma_\beta \frac{1}{\gamma\cdot(P_- - K)-m_e}\gamma_\alpha\right] \\ & \times U(-P_+,S_+) \end{aligned} \tag{12}$$

[11] While this part of the computation was being completed we learned that similar work has been done by G. Oppo and S. Oneda, Phys. Rev. **160**, 1397 (1967). They obtained $\Gamma(\eta \to \pi\gamma\gamma)=0.96$ eV. The slight numerical difference is understandable in view of the ambiguity in identifying the coupling constant with the $M1$ transition amplitude in the quark model.

[12] It will be considerably more difficult to compute the corresponding dispersive amplitude as it is complicated by the ultraviolate divergence. Indeed we may look upon the vector-meson intermediate states in the VMD model as a natural cutoff mechanism.

[13] The advantage of studying the absorptive part of the interaction with a $P_\alpha P_\beta F_\alpha F_{\beta\mu}$ vertex was first pointed out by S. Berman in a private conversation with G. Feinberg, who subsequently informed me.

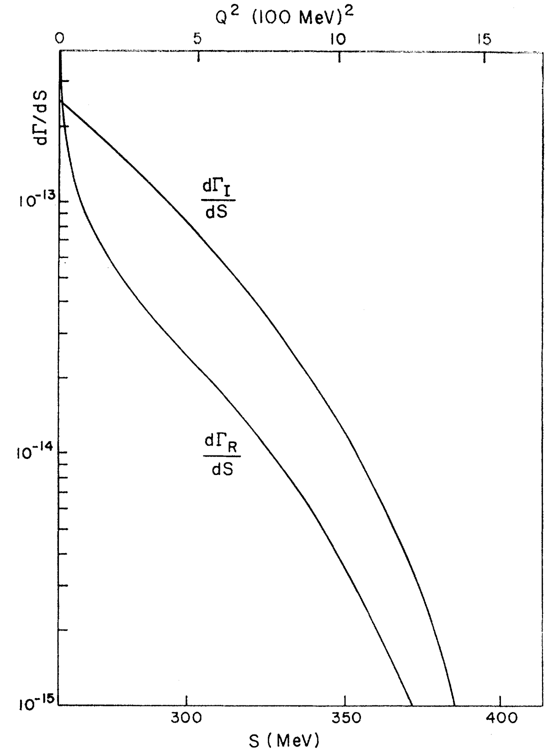

FIG. 3. The $\eta \to \pi e^+ e^-$ decay spectra, $d\Gamma_R/dS$ and $d\Gamma_I/dS$.

again setting $m_e=0$ throughout, then

$$\begin{aligned} M_I(\pi e^+ e^-) &= g\frac{\alpha}{4\pi}\bar{U}(\gamma\cdot P)U \int d^4K\, \delta(K^2)\delta((K-Q)^2) \\ &\quad \times (KQ)\left[\frac{P(K-P_+)}{(KP_+)} - \frac{P(K-P_-)}{(KP_-)}\right] \\ &= \tfrac{1}{4}\alpha g \bar{U}(\gamma\cdot P)U[P(P_+ - P_-)]. \end{aligned} \tag{13}$$

We note that Eq. (13) also agrees with the vector term of Eq. (1). The decay spectrum $d\Gamma_I/dS$ obtained in this way shows a similar Q^2 dependence to that found in the VMD model. For $\eta \to \pi\gamma\gamma$, we have the decay amplitude,

$$M(\pi\gamma\gamma) = 2g(k\cdot p)(q\cdot p)(\epsilon\cdot\epsilon'), \tag{14}$$

where we have chosen the polarization vectors such that they are spacelike in the rest frame of η. In this way we obtain the ratio $\Gamma_I(\pi e^+ e^-)/\Gamma(\pi\gamma\gamma) \approx 10^{-6}$, which is to be compared with the value of $\approx 7\times 10^{-6}$ found in the VMD model [see Eq. (9)]. Allowing for such model-dependent variations, we expect the interference effect with the C-conserving background not to be an important factor in the search for the C-violating $\eta \to \pi e^+ e^-$ until the ratio $\Gamma(\eta \to \pi e^+ e^-)/\Gamma(\eta \to \pi\gamma\gamma)$ is experimentally set at some value less than 10^{-4}.

Note added in proof. Recent experimental results[14] indicate that the rate for $\eta \rightarrow \pi^0\gamma\gamma$ computed in VMD model is too small by an order of magnitude. This may be due to the fact that the VMD amplitude involves higher powers of momenta than are required by gauge invariance.[15] Nevertheless, since the amplitude given by pure S-wave coupling $(F_{\mu\nu}F_{\mu\nu}\phi_\eta\phi_\pi)$ for $\eta \rightarrow \pi^0 e^+e^-$ is proportional to electron mass, the vector-meson intermediate states will still be dominant for this decay process. If we assume pure S-wave coupling for $\eta \rightarrow \pi^0\gamma\gamma$ and VMD for $\eta \rightarrow \pi^0 e^+e^-$, the branching ratio will then be lowered accordingly by an order of magnitude to $\approx 10^{-6}$.

[14] See, for example, M. Feldman, W. Frati, R. Gleeson, J. Halpern, M. Nussbaum, and S. Richert, Phys. Rev. Letters **18**, 868 (1967).

[15] This was pointed out to me by Professor R. H. Dalitz in a private conversation.

ACKNOWLEDGMENT

I am grateful to Professor G. Feinberg for suggesting this problem to me and for guiding me at all stages of the work.

PHYSICAL REVIEW VOLUME 162, NUMBER 5 25 OCTOBER 1967

High-Energy Sum Rule for Kp Scattering*

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(Received 5 June 1967)

A sum rule of the superconvergent type is obtained for forward, elastic Kp scattering. It connects Regge parameters with an integral over total cross sections. A numerical evaluation is carried out and the Regge parameters thus determined are compared to those obtained from a fit to the high-energy data. Impressive agreement is found.

It was recently pointed out[1] that if an amplitude $f(\nu)$ decreases faster than ν^{-1} at high energies, it satisfies a superconvergent relation

$$\int_{-\infty}^{\infty} d\nu \,\mathrm{Im} f(\nu) = 0\,. \tag{1}$$

Subsequently, Logunov *et al.*[2] and Igi and Matsuda[3] have shown that it is possible to write down an analog of relation (1) for an amplitude which is not convergent but whose high-energy behavior is given. Briefly, the procedure consists in writing the given amplitude as a sum of two pieces, one of which is nonconvergent and the other is convergent enough to satisfy the condition of superconvergence. One then writes a superconvergence relation for this latter piece which, of course, is the difference of the given amplitude and its (known) nonconvergent piece. These authors applied the above procedure to the case of forward pion nucleon scattering and obtained a striking agreement with experiment. For a further examination of the underlying assumptions and usefulness of this procedure, we have analyzed the case of forward Kp scattering. We present here an account of this calculation. We find that the results obtained from our sum rule agree very well with the experimental data available at present.

We consider the forward $K^{\pm}p$ elastic scattering amplitude $f^{\pm}(\nu)$ defined as [4]

$$f^{(\pm)}(\nu) = \frac{1}{4\pi}[A(K^{\pm}p) + \nu B(K^{\pm}p)]\,. \tag{2}$$

Since we need a crossing even absorptive part of an amplitude for a superconvergent relation, we introduce the combination

$$F_-(\nu) = \tfrac{1}{2}[f^{(-)}(\nu) - f^{(+)}(\nu)]\,. \tag{3}$$

The absorptive part of this amplitude satisfies the crossing property

$$\mathrm{Im}F_-(-\nu) = +\mathrm{Im}F_-(\nu)\,.$$

Following the method described above, we decompose $F_-(\nu)$ as a sum of a nonconvergent part and a convergent one. For simplicity, we assume that the nonconvergent part can be represented by a sum of Regge-pole terms. We have

$$F_-(\nu) \equiv \sum_i F^{(i)}(\nu) + \epsilon_-(\nu)\,, \tag{4}$$

* Supported in part by the National Research Council of Canada.

[1] L. D. Soloviev, Joint Institute for Nuclear Research Report No. E-2343, Dubna, 1965 (unpublished); V. de Alfaro, S. Fubini, G. Furlan, and G. Rossetti, Phys. Letters **21**, 576 (1966).

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