FIXED POLES IN VIRTUAL COMPTON AMPLITUDES

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It is suggested that the leading fixed-pole terms in virtual Compton amplitudes are polynomial functions of the photon mass variable q^2. Several arguments are given in support of this conjecture. Implications of this behavior in the calculation of the electromagnetic mass differences and in the q^2 dependence of the e-p inelastic-scattering structure functions are discussed.

It has been suggested on theoretical grounds that certain weak amplitudes should have fixed poles in the complex angular momentum (J) plane at both right- and wrong-signature nonsense points.[1,2] This result is supported by numerical studies of various sum rules[3] and, recently, by a detailed analysis of the forward Compton amplitude as constructed from the observed total photoabsorption cross sections via dispersion relations.[4] In this note, we study the dependence of fixed-pole terms in Compton amplitudes on the virtual-photon mass variable (q^2).

Consider the forward spin-averaged virtual Compton amplitude

$$T_{\mu\nu}{}^{\alpha\beta}=i\int d^4x\, e^{-iqx}\langle p|T^*[J_\mu{}^\alpha(x)J_\nu{}^\beta(0)]|p\rangle, \tag{1}$$

where α,β are isospin indices. We shall denote amplitudes symmetric and antisymmetric in α,β by the superscripts + and −, respectively. Each of these amplitudes may be separated into two parts:

$$T_{\mu\nu}{}^{(\pm)}=S_{\mu\nu}{}^{(\pm)}+R_{\mu\nu}{}^{(\pm)}, \tag{2}$$

where $S_{\mu\nu}{}^{(\pm)}$ are the "singular" parts in the low-energy limit ($q^\mu\to 0$) and satisfy the Ward identities: $q^\mu S_{\mu\nu}{}^{(-)}=p_\nu$ and $q^\mu S_{\mu\nu}{}^{(+)}=0$ (for all values of q); whereas $R_{\mu\nu}{}^{(\pm)}$ vanish in the limit $q^\mu\to 0$ and satisfy $q^\mu R_{\mu\nu}{}^{(\pm)}=0$. The bare Born term satisfies the two requirements on $S_{\mu\nu}$ and thus explicitly

$$S_{\mu\nu}{}^{(-)}=[4(q\cdot p)p_\mu p_\nu-q^2(q_\mu p_\nu+p_\nu q_\nu)+(q\cdot p)q_\mu q_\nu]/(4m^2\nu^2-q^4), \tag{3}$$

$$S_{\mu\nu}{}^{(+)}=2[q^2p_\mu p_\nu-(q\cdot p)(q_\mu p_\nu+p_\mu q_\nu)+(q\cdot p)^2g_{\mu\nu}]/(4m^2\nu^2-q^4), \tag{4}$$

$$R_{\mu\nu}{}^{(\pm)}=t_1{}^{(\pm)}(\nu,q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)+t_2{}^{(\pm)}(\nu,q^2)[q^2p_\mu p_\nu-(q\cdot p)(q_\mu p_\nu+p_\mu q_\nu)+(q\cdot p)^2g_{\mu\nu}], \tag{5}$$

where $\nu=-q\cdot p/m=(s-u)/4m$. The invariant functions $t_1{}^{(\pm)}$ and $t_2{}^{(\pm)}$ are free of kinematic singularities and zeros. The regularized t-channel double-helicity-flip amplitudes [essentially the coefficients of the $p_\mu p_\nu$ terms in Eqs. (3)-(5)] are

$$\tilde{f}_{1,-1}{}^{(-)}=mq^2t_2{}^{(-)}-\frac{\nu}{\nu^2-q^4/4m^2}, \tag{6}$$

$$\tilde{f}_{1,-1}{}^{(+)}=mq^2t_2{}^{(+)}+\frac{q^2}{2m(\nu^2-q^4/4m^2)}. \tag{7}$$

We note that the assumption[5] that $t_2{}^{(-)}$ is Regge behaved (thus superconvergent in this case) implies a fixed pole at $J=1$ in $\tilde{f}_{1,-1}{}^{(-)}$ with the residue equal to 1. Using the Froissart-Gribov formula, one obtains

$$(2/\pi)\int \mathrm{Im}\tilde{f}_{1,-1}{}^{(-)}(\nu,q^2)d\nu=1. \tag{8}$$

This is the $t=0$ current-algebra sum rule of Adler, Dashen, Gell-Mann, and Fubini.[1] A similar assumption that $t_2{}^{(+)}$ is Regge behaved leads to a $J=0$ fixed pole in $\tilde{f}_{1,-1}{}^{(+)}$ with a residue function given by the classical Thomson amplitude. Strong evidence for such a term at $q^2=0$ has been found by Damashek and Gilman.[4]

These discussions suggest that fixed poles, being a special feature of localized current-current interactions, are closely associated with the "singular" parts of the amplitude which are insensitive to the structure arising from strong interactions. A particularly interesting consequence of this observation is the following Ansatz: The residue functions of these leading right-signatured fixed poles are independent (or simple polynomials) of the virtual photon mass variable q^2. In what follows we shall give supporting arguments to this conjecture and consider its implications for the electromagnetic mass differences and for the q^2 dependence of the e-p inelastic-scattering structure functions.

For simplicity, we will first consider the $J=1$ fixed pole in $\tilde{f}_{1,-1}{}^{(-)}$. Without making use of current algebra, the residue is a general function of q^2 [cf. Eq. (8) and footnote 5],

$$R(q^2)=(2/\pi)\int \mathrm{Im}\tilde{f}_{1,-1}{}^{(-)}(\nu,q^2)d\nu. \tag{9}$$

We assume that $R(q^2)$ satisfies a dispersion relation[6,7] in q^2,

$$R(q^2)=\sum_{n=1}^{N}(q^2)^{n-1}R_n+\frac{(q^2)^N}{\pi}\int dq'^2\frac{\mathrm{Im}R(q'^2)}{(q'^2)^N(q'^2-q^2)} \tag{10}$$

with a finite number (N) of subtractions. Bearing in mind the fact that $\mathrm{Im}\tilde{f}_{1,-1}$ is the matrix element of a product of two currents [cf. Eq. (1)], we can graphically represent the right-hand side of Eq. (10) as in Fig. 1. There, a cross at the end of a photon line indicates possible subtractions in q^2 (contact terms). Figure 1(a) corresponds to the subtraction term in Eq. (10) while Figs. 1(b)-1(d) all contribute to the dispersion integral. The contributions from Figs. 1(b) and 1(c) are proportional to the residue functions of a fixed pole at $J=1$ for photoproduction amplitudes, and from Fig. 1(d) for a hadron-hadron scattering amplitude. Pure hadronic amplitudes cannot have fixed poles at right-signatured points. There is also evidence, both theoretical[8] and experimental,[9] against the presence of fixed poles for photoproduction of hadrons. The absence of the $J=1$ fixed pole in these amplitudes (or equivalently, the validity of the corresponding superconvergence relations) allows us to conclude that the only surviving contribution to the residue function (9) is the polynomial term in Eq. (10), i.e.,

$$R(q^2)=\sum_{n=1}^{N}(q^2)^{n-1}R_n. \tag{11}$$

For the case under consideration, current-algebra sum rule Eq. (8) verifies this behavior with $N=1$ and $R(q^2)=1$.

The same considerations can be applied to $\tilde{f}_{1,-1}{}^{(+)}$. However, here the fixed-pole contribution is not expected to be the leading term in the high-energy limit. There are Regge poles lying above the point $J=0$ at $t=0$. The Froissart-Gribov formula cannot be naively used down to $J\simeq 0$ to give us a representation of the residue corresponding to that of Eq. (9). This situation can be saved by considering the amplitude with its leading Regge contributions subtracted (in the sense of the finite-energy sum rule). For the reduced amplitude, the Froissart-Gribov definition is valid down to $J\simeq 0$ and the arguments presented in the previous paragraph may be carried through. One reaches the same conclusion about the q^2 dependence of $R(q^2)$ as before.

Next, consider the field-theory model of Bronzan et al.[1] It consists of self-coupled scalar particles Thus, except for the isospin indices, it is essentially the φ^3 theory. Without going into details (which can be found in Ref. 1) we write the t-channel ($\gamma_1\gamma_2\to\varphi\varphi$) partial-wave amplitudes in the following form:

$$F_{\lambda_1\lambda_2}{}^J(t,q_1{}^2,q_2{}^2,m^2,m^2)=I_{\lambda_1\lambda_2}{}^J(t,q_1{}^2,q_2{}^2,m^2,m^2)$$
$$+\int dk_1{}^2dk_2{}^2I_{\lambda_1\lambda_2}{}^J(t,q_1{}^2,q_2{}^2,k_1{}^2,k_2{}^2)\rho^{-1}(m^2,k_1{}^2,k_2{}^2)T^J(t,k_1{}^2,k_2{}^2,m^2,m^2), \tag{12}$$

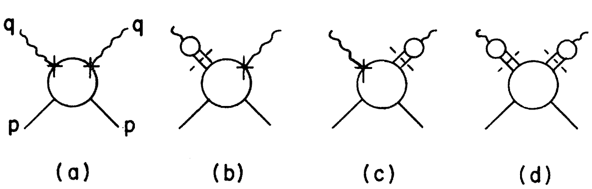

FIG. 1. Unitary diagrams in the q^2 variable for $R(q^2)$.

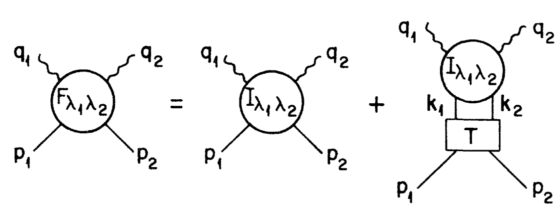

FIG. 2. Virtual Compton amplitudes in the field-theory model, Eq. (12).

where $(\lambda_1\lambda_2)$ are helicities of the photons; $I_{\lambda_1\lambda_2}{}^J$ consists of all the irreducible graphs; T^J is the amplitude for (off-shell) $\varphi\varphi$ scattering; and ρ^{-1}, aside from some trivial normalization factors, is the product of two propagator functions of the intermediate φ's. This equation and the relevant kinematics are illustrated in Fig. 2. Since the right-hand side of Eq. (12) is linear in $I_{\lambda_1\lambda_2}{}^J$, any fixed singularity of $I_{\lambda_1\lambda_2}{}^J$ which is not shared by T^J will show up in the full amplitude $F_{\lambda_1\lambda_2}{}^J$. Furthermore, since $I_{\lambda_1\lambda_2}{}^J$ is the only q^2-dependent factor on the right-hand side, the q^2 dependence of any fixed-pole term in the full amplitude can be directly inferred from that of the irreducible part $I_{\lambda_1\lambda_2}{}^J$. Thus we will concentrate on the irreducible graphs. Just as in the case of the $J=1$ pole in the isospin-antisymmetric amplitude,[1] the Born diagrams generate a $J=0$ fixed pole in the isospin-symmetric part of $I_{1,-1}{}^J$. This term is independent of $q_1{}^2$ and $q_2{}^2$, and it behaves as s^{-2} for large s. But, unlike the antisymmetric case, a number of the second-order diagrams in $I_{1,-1}$ can also behave[10] asymptotically as s^{-2} and $s^{-2}\ln s$, again reflecting the fact that there are moving poles lying above the $J=0$ point.[11] However, we note that (i) the coefficients of these s^{-2} terms are also independent of q^2 for large s and (ii) these terms do not cancel the Born-diagram contribution. We conclude, therefore, that the $J=0$ fixed pole in $I_{1,-1}{}^J$, and thus also in the full amplitudes $F_{1,-1}{}^J$, is independent of q^2.

The presence of a $J=0$ fixed pole in the isospin-symmetric virtual Compton amplitude has direct consequences for the calculation of the electromagnetic mass differences via the Cottingham formula. The evidence for the presence of this pole at $q^2=0$,[4] together with the lack of success in recent attempted calculations of the n-p mass difference without fixed poles,[12] have led to the belief[13] that such a fixed pole is indeed relevant, at least for the $I=1$ mass differences. However, if the residue function of this $J=0$ fixed pole is a polynomial function of q^2, as suggested here, the contribution of this term to the mass difference must diverge.[12] (The divergence is logarithmic if the q^2 dependence is taken to be that suggested by the bare Born term.) It seems, therefore, barring unforseen cancellations, that the addition of a fixed-pole term is not the solution to the dilemma confronting this approach to the electromagnetic mass-difference problem.

Finally, we discuss briefly the fixed poles at wrong-signatured, nonsense points. A pole at $J=1$ in the isospin-symmetric amplitude $\tilde{f}_{1,-1}{}^{(+)}$ is of this type. In the φ^3 model, since the Born term alone is responsible for the $J=1$ fixed pole in the irreducible part $I_{1,-1}{}^J$, the q^2 dependence of this wrong-signatured pole is again a trivial one.[14] On the other hand, the possible presence of this pole in the corresponding strong amplitudes[15] renders our arguments leading to the vanishing of the dispersion integral in Eq. (1) invalid. We note, however, that even in the absence of wrong-signatured fixed poles in strong amplitudes, the virtual Compton amplitude can still have such poles.[16] This means that the subtraction term in Eq. (10) is in general present; therefore the residue function $R(q^2)$ behaves as a polynomial for large q^2.

The presence of a wrong-signatured pole at $J=1$ for the $I=0$ Compton amplitude is necessary[2] to restore the Pomeranchuk pole contributions to $\tilde{f}_{1,-1}$ at $t=0$, thus maintaining a nonvanishing high-energy photoabsorption cross section, as experimentally observed. The two poles of the partial wave amplitude appear in multiplicative form[2]:

$$\lim_{J\to 1}\left[\frac{\beta(q^2,t)}{(J-1)[J-\alpha_P(t)]}\right]_{t=0}. \qquad (13)$$

According to our arguments, β contains a term which is polynomial in q^2.[17] If we again use the bare Born term as a guide for this q^2 dependence, we can easily conclude that the e-p inelastic-scattering structure function νW_2 (the absorptive part of $\nu\tilde{f}_{1,-1}$) must behave, at large ν and q^2, as a constant in both variables. This agrees with the experimental result.[18]

In conclusion, we have presented several arguments for the polynomial q^2 dependence of fixed pole terms in Compton amplitudes. None of these arguments constitutes a proof of the stated q^2 dependence. However, taken as a whole they do suggest a reasonable physical picture: The fixed poles, being associated with localized interactions of currents, are insensitive to the details of strong interactions and consequently do not have any structure in their q^2 dependence. Applications of this idea to other processes, as well as generalization to other types of singularities (for instance, Kronecker deltas), deserve further investigation.

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[1]J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. 157, 1448 (1967), and references cited therein.

[2]H. D. I. Abarbanel, F. E. Low, I. J. Muzinich, S. Nussinov, and J. H. Schwarz, Phys. Rev. 160, 1329 (1967).

[3]G. C. Fox and D. Z. Freedman, Phys. Rev. 182, 1628 (1969).

[4]M. Damashek and F. J. Gilman, to be published; also M. J. Creutz, S. D. Drell, and E. A. Paschos, Phys. Rev. 178, 2300 (1969).

[5]It is interesting to note that without this assumption Eq. (6) already implies the existence of this fixed pole at $q^2=0$ with residue $R(q^2=0)=1$, t_2 being free of kinematic singularities.

[6]The full amplitude f_{+-} has three sets of singularities generated by unitarity in the channels corresponding to the s, u, and q^2 variables. (These variables are related by $s+u=2m^2-2q^2$.) Because of s-u crossing symmetry, however, the function R receives equal contribution from the s- and u-channel absorptive parts, consequently one can forget about the u-channel contribution in (9). Since the only remaining singularities come from s and q^2 channels, it is reasonable to assume that one only encounters normal thresholds in q^2 when s is integrated over. Rigorous proof of the analyticity properties in q^2 is rather difficult. We have checked that the stated analyticity holds to each order in perturbation theory for t-channel ladder diagrams.

[7]The possible usefulness of q^2-dispersion relations in this context was first suggested to us by R. F. Dashen.

[8]R. F. Dashen and S. Y. Lee, Phys. Rev. Letters 22, 366 (1969), and references cited therein. The argument for absence of fixed poles is not valid if there is an "elementary" spin-1 boson that can couple to the photon. There is, of course, no experimental evidence for such a particle.

[9]D. Tompkins et al., Phys. Rev. Letters 23, 725 (1969).

[10]P. G. Federbush and M. T. Grisaru, Ann. Phys. (N.Y.) 22, 263, 299 (1963); G. Tiktopoulos, Phys. Rev. 131, 480, 2373 (1963).

[11]The precise J-plane nature of the s^{-2} terms from the second-order diagrams cannot be ascertained without the explicit evaluation of the Froissart-Gribov formula for the partial-wave amplitude at large ReJ and continuing down to $J=0$, the formula itself being divergent near $J=0$. For our purposes, however, it is sufficient to know that any such terms are necessarily independent of q^2. (These terms, together with other diagrams, may sum up to give the Regge poles mentioned in the text.)

[12]M. Elitzur and H. Harari, Ann. Phys. (N.Y.) 56, 81 (1970); R. Chanda, Phys. Rev. 188, 1988 (1969); D. J. Gross and H. Pagels, Phys. Rev. 172, 1381 (1968).

[13]See, for example, F. J. Gilman, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969 (unpublished).

[14]The difference between the symmetric and antisymmetric cases lies in the different Regge poles in the strong part T^J in Eq. (12): P, P', A_2 in the former, ρ in the latter.

[15]S. Mandelstam and L. L. Wang, Phys. Rev. 160, 1490 (1967).

[16]That is, there are two independent mechanisms for generating wrong-signatured fixed poles in Compton amplitudes; see Ref. 2.

[17]This is to be contrasted to the contributions from "ordinary" Regge poles. The latter, being closely related to resonance production, have form-factor–type rapid-falloff behavior in q^2. See H. Harari, Phys. Rev. Letters 22, 1078 (1969).

[18]E. D. Bloom et al., Phys. Rev. Letters 23, 930, 935 (1969).