PHYSICAL REVIEW D VOLUME 4, NUMBER 5 1 SEPTEMBER 1971

Low-Energy Photoproduction and the Chiral-Transformation Property of the Electromagnetic Current

T. P. Cheng* and Roger Dashen†
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 14 May 1971)

Using new multipole analyses of low-energy pion photoproduction, we have examined the possibility that the electromagnetic current does not commute with the axial charge F_3^5. Within errors, which are not as small as one would like, there is no evidence for a lack of commutation. It is pointed out that the photoproduction amplitude which *directly* measures this commutator receives very little contribution from the large, well-determined M_{1+} multipole and is dominated by the more exotic multipoles like E_{1+}.

In Gell-Mann's SU(3) ⊗ SU(3) algebra of currents, the assumption that the hadronic electromagnetic current transforms as a U-spin singlet member of the $(1,8)\oplus(8,1)$ representation leads to the following commutation relation[1]:

$$[F_a^5(x^0), J_\lambda^{\mathrm{EM}}(x^0,\vec{x})]=0, \quad a=3,6,7, \text{ or } 8. \tag{1}$$

That is, the electromagnetic current commutes with an electrically neutral axial charge. The purpose of this note is to discuss several questions concerning the validity of this commutation relation. A related, although rather different, analysis has recently been carried out by Weinstein.[2]

In the applications of current algebra there are a number of outstanding difficulties that could possibly be traced to the vanishing of the commutator in (1).

First of all, there is the $\eta\to 3\pi$ puzzle.[3] In the usual picture this decay proceeds via a second-order virtual electromagnetic interaction with amplitude

$$\mathcal{A}(\eta\to 3\pi)\propto\langle 3\pi|\mathcal{H}^{\mathrm{EM}}(0)|\eta\rangle, \tag{2}$$

with

$$\mathcal{H}^{\mathrm{EM}}(x)=\tfrac{1}{2}\int d^4y\, D^{\mu\nu}(x-y)T^*(J_\mu^{\mathrm{EM}}(y)J_\nu^{\mathrm{EM}}(x)), \tag{3}$$

$D_{\mu\nu}(x-y)$ being the photon propagator. Partial conservation of axial-vector current (PCAC) then informs us that in the limit of zero four-momentum q of any one of the neutral pions, the amplitude is related to the matrix element of a commutator:

$$\mathcal{A}(\eta\to 3\pi; q=0)\propto\langle 2\pi|[F_3^5,\mathcal{H}^{\mathrm{EM}}(0)]|\eta\rangle, \tag{4}$$

which vanishes by Eq. (1). Thus current algebra predicts that, to zeroth order in SU(2) ⊗ SU(2) symmetry breaking, this decay process is forbidden.

The second difficulty that we will mention here is the formula,[4] valid in the SU(3) ⊗ SU(3)-symmetric limit, for the electromagnetic mass shifts of the pseudoscalar mesons, π's and K's. The symmetry is assumed to be realized in the "Goldstone mode," i.e., the eight pseudoscalar mesons are massless. It then follows that in the presence of a symmetry-breaking perturbation [in this case the $\mathcal{H}^{\mathrm{EM}}$ of Eq. (3)] the mesons acquire masses

$$m_{ab}^2=\langle a|\mathcal{H}^{\mathrm{EM}}(0)|b\rangle+O(\alpha^2).$$

Again using the standard PCAC technique, we relate it to a double commutator:

$$m_{ab}^2=-\langle 0|[F_a^5,[F_b^5,\mathcal{H}^{\mathrm{EM}}(0)]]|0\rangle+O(\alpha^2,\epsilon\alpha). \tag{5}$$

This and Eq. (1) give the result that to first order in the fine-structure constant α and zeroth order in the strong symmetry-breaking parameter ϵ, the electromagnetic self-energies of the neutral mesons π^0 and K^0 are zero. The usual assumption that mass splittings in a given isospin multiplet are purely electromagnetic in origin and U-spin invariance then lead to the formula

$$m_{K^+}^2-m_{K^0}^2=m_{\pi^+}^2-m_{\pi^0}^2. \tag{6}$$

The above two results are clearly in violent disagreement with experiments. Among possible resolutions is the possibility that Eq. (1) does not hold in nature; namely, in the electromagnetic current there exist, besides the $(1,8)\oplus(8,1)$ octet piece, extra terms. These could be, for example, an SU(3) singlet or a member of a $\underline{27}$ (including possible isotensor currents) or even an $\underline{8}$ that does not belong to the $(1,8)\oplus(8,1)$. Clearly, in order to resolve the above-mentioned difficulties, the "anomalous" currents must have a magnitude comparable to the normal one. Consequently, a typical matrix element of the commutator in Eq. (1) would have to differ significantly from zero. In the following we shall discuss the present experimental constraint that may be placed upon such a possibility.

Before proceeding to the main topic of this paper, however, two possible sources of confusion should be mentioned. First, we are not talking about the anomalies found by Adler and by Bell and Jackiw,[5] which may be operating in $\pi^0 \to \gamma\gamma$. This type of anomaly can appear only in processes involving two or more (real or virtual) photons and will not explain the puzzles mentioned above. More explicitly, we are looking for a violation of Eq. (1) in the strong interactions alone. Secondly, we will assume that any part of J_λ^{EM} which fails to commute with F_3^5 has the usual charge-conjugation properties. Thus, we are ignoring a possible class of theories where the failure of Eq. (1) is related to C violation.[6,7]

Consider the matrix element of the commutator in Eq. (1) with $a=3$ between proton states of momenta p_1 and p_2:

$$\langle p_2|[F_3^5, J_\lambda^{\text{EM}}(0)]|p_1\rangle \equiv eg_A(\zeta/2M)\bar{u}(p_2)\gamma_5\sigma_{\lambda\mu}k^\mu u(p_1), \tag{7}$$

with

$$k=p_2-p_1 \quad\text{and}\quad k^2=0.$$

Equation (1) demands that $\zeta=0$. We note that ζ is directly related to a certain linear combination of invariant amplitudes for the photoproduction of a neutral pion,

$$\gamma(k)+p(p_1)\to\pi^0(q)+p(p_2),$$

although, at the "soft-pion point," $q=0$. Thus, given the results of multipole analyses, an approximate evaluation of ζ is possible. We shall follow the standard Chew-Goldberger-Low-Nambu notation[8] and denote the four invariant amplitudes as A, B, C, and D. They are functions of $\nu=(p_1+p_2)\cdot q/2M$, $\nu_B=q\cdot k/2M$, and q^2. We shall also make the separation $A=A_B+\tilde{A}$, etc., where A_B is the contribution of the Born-approximation diagrams to the invariant amplitude A. Thus, for example,

$$\begin{aligned} A_B &= -\frac{eg_r}{M}\frac{\nu_B}{\nu_B{}^2-\nu^2}, \\ C_B &= \frac{\kappa_p}{2M}\frac{eg_r}{M}\frac{\nu}{\nu_B{}^2-\nu^2}, \end{aligned} \tag{8}$$

where g_r is the pion-nucleon coupling constant and κ_p is the proton anomalous magnetic moment (measured in nuclear magnetons, $e/2M$; $\kappa_p=1.79$).

Reduction of the final neutral pion in the amplitude

$$\epsilon^\lambda\int d^4x\, e^{-iqx}(-\Box_x+\mu^2)\langle p_2|T(\varphi_3(x)J_\lambda^{\text{EM}}(0))|p_1\rangle$$

and a straightforward application of PCAC leads, in the $q\to 0$ limit, to the result[9]

$$\tilde{A}(\nu=\nu_B=q^2=0)-\frac{\kappa_p}{2M}\frac{eg_r}{M}=-\frac{\zeta}{2M}\frac{eg_r}{M}. \tag{9}$$

(Other non-Born amplitudes vanish in this limit.)

In 1966 Adler and Gilman[10] had already attempted a numerical evaluation of the isospin-even amplitude $A^{(+)}$ at $\nu=\nu_B=0$. Our reasons for coming back to this problem are twofold: At the time of their work, the low-energy photoproduction experiments had not advanced to the degree that detailed model-independent multipole analyses could be made. Thus, in the dispersion integrals, Adler and Gilman kept only multipoles which resonate around the $N^*(1238)$ and the $N^*(1520)$, and the nonresonant S wave. Since their investigation, the amount and quality of data have greatly improved, to the extent that energy-independent multipole analyses are at last being made.[11,12] This provides us with relatively reliable knowledge of a number of the nonresonant multipoles, each of which also makes, as we shall see, important contributions to the dispersion integrals. Secondly, we would like to point out that there is a linear combination of the invariant amplitudes $A+\nu C$ ($\equiv T$) which, at the soft-pion point, is *directly* related to the parameter ζ [see Eqs. (8) and (9)][13]:

$$T(\nu=\nu_B=q^2=0)=-\frac{\zeta}{2M}\frac{eg_r}{M}. \tag{10}$$

Using this amplitude it can be shown that the value of ζ is insensitive to the resonant M_{1+} wave; its contribution to T in the low-energy region is kinematically suppressed.[14] Thus for our purpose, the result deduced from Ref. 10 cannot be taken as final since at that time the only reliably known multipole was M_{1+} (which makes the dominant contribution to A, but not to T or ζ). In fact, from a cursory examination it seems that contributions by other multipoles, e.g., E_{0+}, E_{1-}, E_{1+}, and M_{1-}, etc., can easily add up to a value of ζ comparable to κ_p giving a large violation of Eq. (1). There are also some differences of attitude between Ref. 10 and us. While the main concern of Ref. 10 is to test the validity of PCAC, we shall, on the other hand, assume at the outset that PCAC is good to (10–20)% and we only wish to examine the possibility that the commutation relation in Eq. (1) may be incorrect, giving rise to a *large*, say 50%, violation of the ($\zeta=0$) low-energy theorem. Consequently, for our purpose we shall not attempt any calculation of the corrections coming from the pion being off the mass shell, $q^2=0$. But we expect that if $\zeta=0$, the absolute value of T for any values of ν^2, ν_B, q^2 of the order $\mu_\pi{}^2$ should not be significantly greater than $0.015\mu_\pi{}^{-2}$, that is, 20% of

$$\frac{\kappa_p}{2M}\frac{eg_r}{M}\approx 0.082\,\mu_\pi{}^{-2}\quad(\text{for } g_r{}^2/4\pi\approx 14.6).$$

The multipoles used for our calculation are those of Berends and Weaver,[12] who have made an

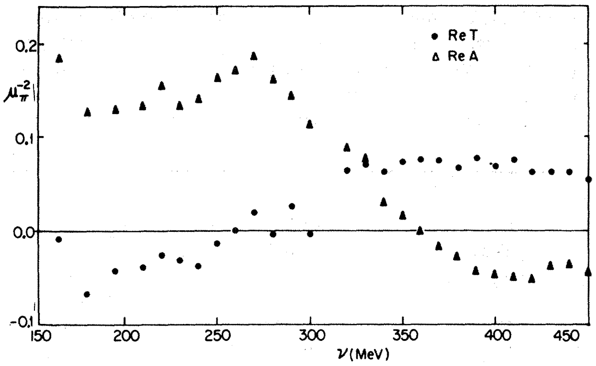

FIG. 1. Real parts of amplitudes A and T at $\nu_B = 0$ as computed from multipoles of Berends and Weaver.

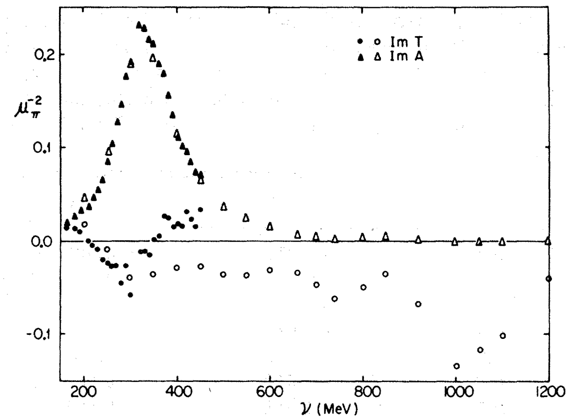

FIG. 2. Imaginary parts of amplitudes A and T at $\nu_B = 0$ as computed from multipoles of Berends and Weaver (solid symbols) and Walker (open symbols).

energy-independent analysis of pion photoproduction off a proton at 27 energies below the photon lab energy of 450 MeV. These authors used all existing data, and a continuous solution was found. This was then supplemented at higher energies (up to 1200 MeV) by the older and energy-dependent solution of Walker.[15]

Figures 1 and 2 display the real and imaginary parts of the amplitudes (at $\nu_B = 0$)[16] computed from multipoles. That ζ could significantly differ from zero is indicated by the values of amplitudes, $T \approx -0.06\mu_\pi^{-2}$, $A \approx 0.12\mu_\pi^{-2}$, at $\nu = 1.1-1.3\mu_\pi$. However, we should not take this too seriously. The values of multipoles are not reliable at these extremely low energies (below 200 MeV) where polarization and asymmetry data still do not exist and differential cross-section data are not abundant.

In the following, fixed-t dispersion relations will be used to compute values of amplitudes at $\nu = \nu_B = 0$. Even though A and T are expected to satisfy unsubtracted dispersion relations,[17] we shall still make a subtraction so that our final results will be almost completely determined by the multipoles at energies around 200–400 MeV, where their values are the most reliable.[18] So that our final result will not be overly sensitive to errors in multipoles at any one point, we make the "broad-area" subtraction by placing an artificial cut from $\nu_1 = 210$ MeV to $\nu_2 = 270$ MeV:

$$T(0,0) = \frac{\nu_1^{2\beta}\nu_2^{2(1-\beta)}}{\pi}\left(\int_{\nu_0^2}^{\nu_1^2}\frac{d\nu'^2}{\nu'^2}\frac{\operatorname{Im}T(\nu',0)}{(\nu_1^2-\nu'^2)^\beta(\nu_2^2-\nu'^2)^{1-\beta}} + \int_{\nu_1^2}^{\nu_2^2}\frac{d\nu'^2}{\nu'^2}\frac{\operatorname{Re}T(\nu',0)\sin\beta\pi + \operatorname{Im}T(\nu',0)\cos\beta\pi}{(\nu'^2-\nu_1^2)^\beta(\nu_2^2-\nu'^2)^{1-\beta}}\right.$$
$$\left. - \int_{\nu_2^2}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\operatorname{Im}T(\nu',0)}{(\nu'^2-\nu_1^2)^\beta(\nu'^2-\nu_2^2)^{1-\beta}}\right), \tag{11}$$

where $\nu_0 = \mu_\pi + \mu_\pi^2/2M$ and β is a parameter $(0 \leqslant \beta \leqslant 1)$. A similar dispersion relation holds for $A(0,0)$. A subtraction made in this form also spares us the problem of evaluating principal-value integrals, and thus makes the numerical computation easier. In Table I results of the calculation are shown for various values of β. The subtraction cut is placed low enough so that the last integral in Eq. (11) is rapidly convergent. In practice we have cut it off at $\nu = 1200$ MeV, but actually the contributions from the high-energy region ($\nu > 450$ MeV), where Walker's multipoles have to be used, are negligible; they are typically $+0.003\mu_\pi^{-2}$ (Ref. 19) for T and $-0.002\mu_\pi^{-2}$ for A, and thus well below the over-all uncertainties expected in the final results. The variation of results with β gives us some idea of the compatibility of the multipoles used with respect to dispersion relations. This amount of uncertainty ($\approx 0.01\mu_\pi^{-2}$) is also indicated by the difference between two sides of the kinematic equality:

$$A(0,0) - T(0,0) = \frac{\kappa_p}{2M}\frac{eg_r}{M}. \tag{12}$$

With this uncertainty and the 10–20% PCAC correction in mind, we can conclude that the present photoproduction data are compatible with (but do not require) Eq. (1) and $\zeta = 0$.

The contributions to the dispersion integrals from each of the multipoles are shown in Table II.

TABLE I. Values of π^0 photoproduction amplitudes (in units of μ_π^{-2}) at $\nu=\nu_B=0$ computed for various values of β and $\nu_1=210$ MeV and $\nu_2=270$ MeV in Eq. (11). [If Eq. (1) holds and PCAC corrections are negligible, the expected values of $T(0,0)$ and $A(0,0)$ are 0 and 0.082, respectively.]

β	$T(0,0)$	$A(0,0)$
0.1	−0.019	0.067
0.2	−0.014	0.073
0.3	−0.011	0.077
0.4	−0.010	0.080
0.5	−0.009	0.081
0.6	−0.010	0.082
0.7	−0.011	0.081
0.8	−0.012	0.080
0.9	−0.015	0.078

It is clear that the smallness of $T(0,0)$ is the result of a cancellation among a number of multipoles, each of which makes important contributions. It would certainly be desirable to have a more accurate determination of the nonresonant multipoles which make up the bulk of T. If T were really known to (say) 10%, one would be able to make a much more precise statement about the possibility that ζ is nonzero.

TABLE II. Multipole contributions to the dispersion integral with a broad-area subtraction ($\beta=0.5$). [If Eq. (1) holds, the expected totals for $T(0,0)$ and $A(0,0)$ are 0 and 0.082, respectively.]

	$T(0,0)$	$A(0,0)$
E_{0+}	−0.022	−0.019
E_{1+}	−0.023	−0.011
E_{2-}	0.041	−0.007
E_{2+}	−0.001	0.000
E_{3-}	−0.008	0.001
E_{3+}	0.000	0.000
M_{1-}	−0.004	0.033
M_{1+}	0.010	0.092
M_{2-}	−0.001	−0.001
M_{2+}	−0.003	−0.009
M_{3-}	0.000	0.000
M_{3+}	0.001	0.002
Total	−0.009	0.081

*Research sponsored by the National Science Foundation Under Grant No. GP-16147.

†Alfred P. Sloan Foundation Fellow.

[1]See, for example, S. L. Adler and R. F. Dashen, *Current Algebra and Applications to Particle Physics* (Benjamin, New York, 1968). We shall follow the notations and conventions used in this reference.

[2]M. Weinstein, New York University Report No. 5/71 (unpublished).

[3]D. G. Sutherland, Phys. Letters 23, 384 (1966).

[4]R. Dashen, Phys. Rev. 182, 1245 (1969).

[5]S. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969).

[6]Such a theory has been considered by S. Adler [Phys. Rev. Letters 18, 519 (1967)] in connection with the $\eta\to 3\pi$ puzzle.

[7]If the part of $J_\lambda^{\rm EM}$ which fails to commute with F_3^5 is C-even, then the parameter ζ in Eq. (7) is necessarily zero.

[8]G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

[9]S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento 40, 1171 (1965).

[10]S. Adler and F. Gilman, Phys. Rev. 152, 1460 (1966).

[11]P. Noelle, W. Pfeil, and D. Schwela, Bonn University Report No. 2-79, 1970 (unpublished). We note that the result of this analysis is in general agreement with that of Ref. 12, which is used in our calculation.

[12]F. A. Berends and D. L. Weaver, CEA report, 1970 (unpublished). We are grateful to these authors for sending us a deck of punched cards containing the result of their analysis.

[13]By $\nu=\nu_B=0$, we shall always mean $\nu_B=0$ first, then $\nu=0$.

[14]This suppression of the M_{1+} contribution to the amplitude $A+\nu C$ can be easily demonstrated by an explication calculation in the static limit.

[15]R. L. Walker, Phys. Rev. 182, 1729 (1969).

[16]We have also examined amplitudes at $4M\nu_B=\mu_\pi^2$ and $2\mu_\pi^2$ and have found no major changes.

[17]This is indicated by the Regge-pole analysis given by G. Zweig, Nuovo Cimento 32, 689 (1964).

[18]It is clear that for photoproduction multipoles, the real parts are just as well determined as the imaginary parts.

[19]This value is probably very misleading in view of the large discrepancies between Berends and Weaver (Ref. 12) and Walker (Ref. 15) in values of Im$T(\nu,0)$, as shown in Fig. 2.