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Coincidence Electroproduction and Scaling in the Regge Region

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We consider the coincidence electroproduction process $l + p \to l' + p'$ + anything. With the momentum transfer between p and p' held fixed, a single trajectory in the $p\bar{p}'$ channel dominates as the laboratory energy of the virtual photon becomes large. (It is known that the onset of single-trajectory dominance may be ascertained by observing the azimuthal angular distribution of the detected hadron.) In the region of large virtual photon mass and missing mass (with their ratio fixed) the process may be picturesquely described as "deep-inelastic scattering of a lepton on a Reggeon target." We implement the assumed Regge behavior in the language of van Hove. The scaling behavior of the various structure functions is then determined by the "canonical" light-cone commutator (taken between spin-J states), if certain smoothness assumptions are granted. Relations of the Callan-Gross type are also derived. The experimental features corresponding to our results are relatively easy to observe. The kinematics of coincidence electroproduction (with a polarized beam) is discussed in some detail.

I. INTRODUCTION

A great deal of effort[1] is being devoted to high-energy electroproduction experiments in which a single hadron is detected in coincidence with the outgoing lepton (Fig. 1), i.e.,

$$l + p \to l + \text{hadron} + \text{anything}. \tag{1.1}$$

Such knowledge about the hadronic final states should be indispensable in our attempts to understand the surprising results of the SLAC single-arm experiments

$$e + p \to e + \text{anything}. \tag{1.2}$$

A number of authors[2] have constructed dynamical models to describe process (1.1).

In the one-photon-exchange approximation the dependences on two of the six variables in the cross section for (unpolarized) reaction (1.1) are explicitly given by lepton quantum electrodynamics, thus allowing a simple decomposition in terms of four invariants. These four structure functions are functions of the four remaining dynamical variables. This rich kinematical content provides us with the opportunity of considering a variety of new asymptotic limits. In this paper we will discuss the possibility of relating, in a certain kinematic domain, the cross section of (1.1) to that of a process which may be termed picturesquely as "inelastic lepton-Reggeon scattering." Namely, in the limit of high energy with momentum transfer between the target proton (p) and detected hadron (p') fixed, the electron may be viewed as scattering inelastically off the Reggeon which is expected to dominate in the $p\bar{p}'$ channel (Fig. 2). The Regge limit for this process was first discussed by Pais and Treiman.[3]

The structure functions of the Reggeon measured in this way are related to the absorptive amplitudes of forward Compton scattering on a Reggeon "target" (Fig. 3). We shall then speculate on the possible scaling properties of these novel quantities in the deep-inelastic limit. Rather than appealing to specific dynamical models, we operate on the assumption that all relevant functions are as analytic and smoothly behaved as possible. What this means precisely will be spelled out in detail in the course of our discussion. Briefly we assume that it is meaningful to visualize the Reggeon as a superposition of the effects of exchanging a series of particles of increasing spin (in the manner of van Hove[4]). Thus we consider simply inelastic electron scattering off targets of spin J. To describe the deep-inelastic region we rely on the structure of the light-cone commutator[5] as given by the quark (and the formal quark-gluon[6]) model. Proceeding thusly, we are able to predict the scaling behavior of the various structure functions [see Eq. (4.9) below]. Furthermore, the tensor structure of the light-cone commutator leads to two interesting relations between the various structure functions [see Eqs. (4.11) and (4.12) below]. These relations are of the same type as the Callan-Gross relations.[7] (The second of the two relations is, however, less credible. It is correct if the structure of light-cone expansion may be extracted from a free-quark model up to and including the "twist-four"[6] terms.)

One may in principle use process (1.1) to reach the deep-inelastic structure functions of the pion (when the detected hadron is a nucleon) in much the same way that the pion elastic form factor has been inferred by using the virtual pions of the nucleon as targets in the pion electroproduction experiments.[8] This topic has been the subject of a recent article by Sullivan.[9] But, as to be expected, the practical difficulties in such an extrapolation procedure would be overwhelming. The experimental demand by the process considered here will be less extravagant, since we are only interested in the scaling behavior of the leading Regge trajectory and no experimental extrapolation in the momentum transfer variable is required.

In this paper we will focus our attention on the case when the recoil proton (slow in the laboratory frame) is detected so that the leading trajectory would be Pomeranchukon. At least for the immediate future we expect the statistics to be high enough for our purposes only in this case. To improve statistics one may also integrate the data over ν (lab energy of the virtual photon) without losing much interesting information as long as ν is large enough for diffractive scattering to dominate. (We shall see that the question of whether diffraction indeed dominates may be settled fairly unambiguously with the availability of polarized beams.)

In Sec. II the kinematics of the problem is reviewed and relevant structure functions defined. In Sec. III the "ordinary" Regge limit is taken. We would like to emphasize that the amount of Regge theory used here is minimal. Only certain qualitative features, i.e., dominance by crossed-channel exchanges in the high-energy limit, are needed for the subsequent discussion. It is hoped that features eventually extracted may well be more general than the specific details of the Regge-pole model. The deep-inelastic limit is then taken in Sec. IV. Results for the scaling structure functions are obtained. Finally a discussion of our results is presented in Sec. V. An appendix contains some of the kinematical details.

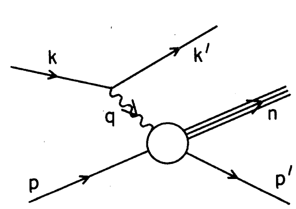

FIG. 1. Coincidence electroproduction (1.1) in the one-photon-exchange approximation.

II. KINEMATICS

Let k, k', p and p' be, respectively, the momenta of the initial and final electrons (muons), and the target proton and the observed final hadron (Fig. 1). We shall consistently ignore the lepton mass and let m and m' be the masses of p and p'. The unobserved hadronic complex has momentum $n=k+p-k'-p'$ and invariant mass $M_x=\sqrt{n^2}$.

We introduce the following combinations:

$$q=k-k', \quad P=p+p', \quad \Delta=p-p'. \tag{2.1}$$

The six variables needed for describing the process (1.1) are chosen to be

$$\psi, \quad \phi, \quad q^2, \quad q\cdot P, \quad q\cdot\Delta, \quad \Delta^2. \tag{2.2}$$

In the laboratory frame

$$\cos\psi=\frac{\epsilon+\epsilon'}{(\nu^2+Q^2)^{1/2}}, \tag{2.3}$$

$$\cos\phi=\frac{\vec{k}\times\vec{k}'}{|\vec{k}\times\vec{k}'|}\cdot\frac{\vec{q}\times\vec{p}'}{|\vec{q}\times\vec{p}'|}, \tag{2.4}$$

where ϵ (ϵ') and ν are energies of the initial (final) lepton and the virtual photon, respectively, and $Q^2=-q^2$.

In the one-photon-exchange approximation (Fig. 1) the explicit form of the leptonic vertex fixes the dependences on ψ and ϕ. The remaining four variables in (2.2) are needed to describe the hadronic "black box":

$$\gamma_v+p\to\text{hadron}+\text{anything}, \tag{2.5}$$

with corresponding helicity amplitude [for virtual photon γ_v in a given polarization state a (=+, 0, −)],

$$J^{(a)}(n,p';q,p)=\epsilon_\mu^{(a)}\langle(\text{out})n,p'|J^\mu(0)|p\rangle\,. \tag{2.6}$$

This task of factorizing off the known dependence of the amplitude may be effected most transparently in the brick-wall frame and by using the so-called O(2, 1) formalism.[10-12] (Of course, other methods[13] are just as equal to the tasks.) In the Appendix this procedure is given in some detail.

Since the muon beam at the National Accelerator Laboratory comes to us polarized without cost we write down the cross section of process (1.1) for an initial lepton beam with longitudinal polarization ρ:

$$E_{p'}\frac{d\sigma(\rho)}{d^3p'd\nu dQ^2}$$

$$=4m(Q^2+\nu^2)^{1/2}\frac{d\sigma(\rho)}{d\phi d\Delta^2 dM_x^{\,2}d\nu dQ^2}$$

$$=\left(\frac{\alpha}{4\pi}\right)^2\frac{2m'}{\epsilon^2Q^2}\frac{1}{1-x}\{H^{++}+xH^{00}-x\cos 2\phi\, H^{+-}-2[2x(x+1)]^{1/2}\cos\phi\,\mathrm{Re}H^{+0}+\rho 2[x(1-x)]^{1/2}\sin\phi\,\mathrm{Im}H^{+0}\}\,, \tag{2.7}$$

with

$$x=\frac{\cosh^2\psi-1}{\cosh^2\psi+1}=\frac{1}{1+2\dfrac{Q^2+\nu^2}{Q^2}\tan^2(\frac{1}{2}\theta)}\,.$$

The helicity structure functions $\{H^{ab}\}$ are related to the amplitude in Eq. (2.6) by

$$H^{ab}=(2\pi)^3\sum_n\delta(p+q-p'-n)J^{(a)}J^{(b)*}\,, \tag{2.8}$$

and they are functions of the last four variables in (2.2).

The fact that the azimuthal angular distribution has the characteristic form $c_1+c_2\cos\phi+c_3\cos 2\phi+c_4\sin\phi$ in the one-photon-exchange approximation was emphasized by Pais and Treiman,[14] and should be exploited as a check on the purity of the data. We note that the ImH term (with its $\sin\phi$ dependence) is unique to a polarized beam. This remark will be important in Sec. III when we discuss the Regge limit.

The advantage of defining the structure functions as products of helicity amplitudes is that they lead to expressions for angular and spin correlations in the form of a simple matrix product[11, 12] and that positivity conditions, when expressed in terms of them, become obvious (see Appendix). However, for certain theoretical discussions, as in the case of light-cone analysis of scaling (Sec. III), it is often more convenient to use structure functions defined by the invariant decomposition of matrix elements of current products:

$$W_{\mu\nu}=(2\pi)^3\sum_n\delta(p+q-p'-n)\langle(\mathrm{out})n,p'|J_\mu(0)|p\rangle\langle(\mathrm{out})n,p'|J_\nu(0)|p\rangle^*$$

$$=W_1g_{\mu\nu}+W_2\Delta_\mu\Delta_\nu+W_3P_\mu P_\nu+W_4\tfrac{1}{2}(\Delta_\mu P_\nu+\Delta_\nu P_\mu)+W_5\tfrac{1}{2}i(\Delta_\mu P_\nu-\Delta_\nu P_\mu)+\cdots\,, \tag{2.9}$$

where the neglected terms are proportional to q_μ or q_ν. All the invariant structure functions $\{W_i\}$ are real. Their relations to $\{H^{ab}\}$ may be deduced simply by noting $H^{ab}=\epsilon_\mu^{(a)}W^{\mu\nu}\epsilon_\nu^{(b)*}$ and they are given in the Appendix.

III. THE REGGE LIMIT

For some *fixed* missing mass the process (2.5) may be viewed conveniently as a two-body-to-two-body reaction. We shall assume that the usual Regge picture is applicable here.

Thus in the kinematic region of very large energy with Q^2 and the invariant mass of the unobserved hadrons held finite and with momentum transfer between target and the observed hadron also held finite, i.e.,

$$q\cdot P\gg Q^2,\quad q\cdot\Delta,\quad \Delta^2, \tag{3.1}$$

we assume that our process is dominated by Reggeon exchanges in the $p\bar{p}'$ channel (Fig. 2). The quantum numbers of the exchanged trajectories are, of course, determined by those of the observed hadron. The formalism developed below should apply to all possible types of detected hadrons.

The standard Regge theory informs us that the asymptotic behavior of the helicity amplitudes (2.6) is specified by the trajectory functions $\alpha_i(\Delta^2)$ in the form of

$$J^{(a)}(q\cdot P,q\cdot\Delta,Q^2,\Delta^2)\approx\sum_i\beta^{(a)}(\alpha_i,Q^2,\Delta^2,q\cdot\Delta)\times(1\pm e^{i\pi\alpha_i(\Delta^2)})(q\cdot P)^{\alpha_i(\Delta^2)}\,. \tag{3.2}$$

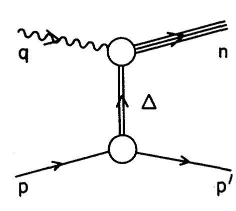

FIG. 2. Reggeon exchanges in the $p\bar{p}'$ channel of (2.5).

To leading order in $(q\cdot P)$ all helicity amplitudes have a common phase determined by the leading trajectory function $\alpha(\Delta^2)$, and, consequently, all helicity form factors become real in this limit.

$$H^{ab}(q\cdot P, q\cdot\Delta, Q^2, \Delta^2) \approx \tilde{H}^{ab}(\alpha, Q^2, \Delta^2, q\cdot\Delta)(q\cdot P)^{2\alpha(\Delta^2)}, \tag{3.3}$$

with

$$\frac{\mathrm{Im}\tilde{H}^{+0}}{\tilde{H}^{ab}} \approx 0$$

for all a and b. For the differential cross-section spectrum (2.7) this implies that the coefficient of (the so-called quasi-T-violating) $\sin\phi$ term should vanish to the leading order in energy. Hence, by observing the azimuthal angle distribution (integrated over the remaining kinematical variables if desired) of the detected hadron, one may be assured that a single trajectory is indeed dominating. This test of single trajectory dominance was first pointed out by Pais and Treiman[3] who emphasized that this feature of Regge theory may have more general validity than the details of the Regge-pole model.

Once one has ascertained that a single trajectory is dominating, one is invited to consider the differential cross section as the absorptive part of the forward Compton amplitudes of virtual photon scattering on a Reggeon (Fig. 3). It is this novel object that we will focus on in Sec. IV.

IV. DEEP-INELASTIC REGGE LIMIT

We shall examine the Q^2 and $q\cdot\Delta$ dependences of this Reggeon Compton amplitude – always with Δ^2 fixed (and small), regarding it as the (mass)^2 of the target Reggeon. We are especially interested in the possible scaling properties these structure functions may exhibit in the deep-inelastic limit, defined as

$$Q^2,\ q\cdot\Delta\to\infty \quad\text{with}\quad \frac{q\cdot\Delta}{Q^2}\ \text{fixed}. \tag{4.1}$$

Namely, we are interested in the subdomain of Regge limit (3.1) in which the virtual photon mass and the missing hadronic mass also grow very large (but, of course, still small compared to ν). We shall refer to this limit as the deep-inelastic Regge limit.

As was mentioned, the operating assumption which empowers us to make nontrivial statements about this deep-inelastic Regge domain is that all relevant functions behave as smoothly as possible. We now explain what we mean by describing our analysis. We visualize the Reggeon, in the manner of van Hove, as a sum of t-channel exchanges of increasing mass and spin[15] (see Fig. 2). By invoking the "canonical" light-cone commutator we determine the behavior of the electroproduction structure functions on spin-J targets in the deep-inelastic domain. We hope that the general scaling behavior thus deduced persists after summing the series.

In this way we are led to consider the following sum[16] for $W^{\mu\nu}$ of Eq. (2.9) [see Fig. 4]:

$$W^{\mu\nu} = \sum_{J,J'} G(J)_{\{\beta\}} D(J)^{\{\beta,\alpha\}} W(J,J')^{\mu\nu}_{\{\alpha,\alpha'\}} \times D(J')^{\{\alpha',\beta'\}} G(J')_{\{\beta'\}}. \tag{4.2}$$

$G(J)_{\{\alpha\}}$ represents the coupling of a spin-J field to the initial and detected hadrons p and p',

$$G(J)_{\{\alpha\}} = g(J,\Delta^2) P_{\alpha_1} P_{\alpha_2}\cdots P_{\alpha_J} + \cdots. \tag{4.3}$$

$D(J)_{\{\alpha,\beta\}}$ represents the propagator of the exchanged spin-J particle,

$$D(J)_{\{\alpha,\beta\}} = [\Delta^2 - M(J)^2]^{-1}\{g_{\alpha_1\beta_1} g_{\alpha_2\beta_2}\cdots g_{\alpha_J\beta_J} + \text{all permutations on } \alpha \text{ and } \beta + \cdots\}. \tag{4.4}$$

(The terms not explicitly displayed contribute to terms of lower order in ν.) $W(J,J')^{\mu\nu}_{\{\alpha,\alpha'\}}$ is the

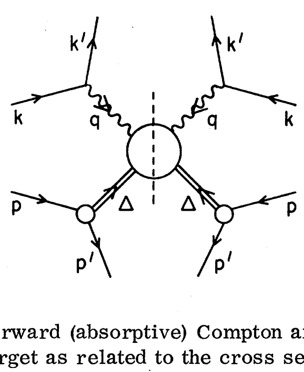

FIG. 3. Forward (absorptive) Compton amplitude on a Reggeon target as related to the cross section of process (1.1) in the Regge limit (3.1).

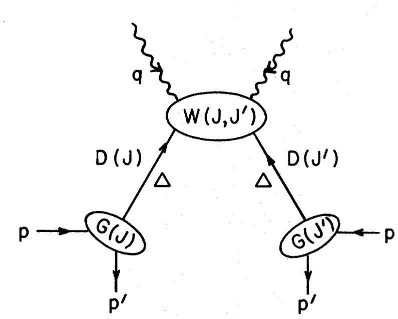

FIG. 4. Spin-J and -J' exchange contribution to cross section of (1.1).

virtual Compton amplitude. When the spin-J and spin-J' particles are on-shell, it is defined by

$$W^{\mu\nu}(J,J') = \epsilon_J^{*\{\alpha\}} W(J,J')^{\mu\nu}_{\{\alpha,\alpha'\}} \epsilon_{J'}^{\{\alpha'\}}$$
$$= \sum_n \langle \Delta, \epsilon_J | J^\mu(0) | n\rangle \times \langle n | J^\nu(0) | \Delta', \epsilon_{J'}\rangle \, \delta^4(q+\Delta-n)\,, \tag{4.5}$$

where $\epsilon_J^{\{\alpha\}}$ denotes the polarization tensor for a particle of spin J. Here $\Delta^2 = M(J)^2$ and $\Delta'^2 = M(J')^2$. Eventually we must continue back to the point of $\Delta = \Delta'$ with Δ^2 negative and small, of course. It is part of our assumption that all such continuations are valid, at least for our purposes. (See discussion in Sec. V.)

The standard argument that light cone dominates in the limit of $Q^2 \to \infty$ and $Q^2/q\cdot\Delta$ fixed can now be applied to the amplitude in Eq. (4.5). We use the formal light-cone commutator extracted from the quark model,[6,17] viz.,

$$[J_\mu(x), J_\nu(0)] \underset{x\approx 0}{\approx} [(g_{\mu\gamma}g_{\nu\sigma} + g_{\nu\gamma}g_{\mu\sigma} - g_{\mu\nu}g_{\gamma\sigma})V^\sigma(0,x) - i\epsilon_{\mu\nu\gamma\sigma}A^\sigma(0,x)]\partial^\gamma[\epsilon(x_0)\delta(x^2)]\,, \tag{4.6}$$

where $V^\sigma(0,x)$ and $A^\sigma(0,x)$ are the vector and axial-vector bilocal operators. Clearly A^σ contributes only to W_5 and hence can be ignored for our purposes. We write down the matrix element of V^σ,

$$\begin{aligned}\langle \Delta, \epsilon_J | V^\sigma(0,x) | \Delta', \epsilon_{J'}\rangle &= -i\tilde f_{J,J'}\Delta^\sigma \epsilon_J^{*\{\alpha\}} x_{\alpha_1}\cdots x_{\alpha_J}\epsilon_{J'}^{\{\alpha'\}} x_{\alpha'_1}\cdots x_{\alpha'_{J'}} \\ &+ \tfrac{1}{2}\tilde g_{J,J'}[\epsilon_J^{*\sigma\alpha_2\cdots\alpha_J} x_{\alpha_2}\cdots x_{\alpha_J}\epsilon_{J'}^{\{\alpha'\}} x_{\alpha'_1}\cdots x_{\alpha'_{J'}} \\ &\qquad + \epsilon_J^{*\{\alpha\}} x_{\alpha_1}\cdots x_{\alpha_J}\epsilon_{J'}^{\sigma\alpha'_2\cdots\alpha'_{J'}} x_{\alpha'_2}\cdots x_{\alpha'_{J'}}] \\ &+ i\tilde h_{J,J'}\Delta^\sigma[\epsilon_J^{*\{\alpha\}}\epsilon_{J'}^{\{\alpha'\}} g_{\{\alpha\}\{\alpha'\}} x\cdots x] \\ &+ (\Delta \leftrightarrow \Delta') + \cdots\,.\end{aligned} \tag{4.7}$$

(In the $\tilde h$ term the notation means that all the indices of ϵ_J are dotted into the indices of $\epsilon_{J'}$ with the remaining indices dotted with x's.) The terms not explicitly displayed do not contribute to leading order in our limit. The bilocal form factors $\tilde f$, $\tilde g$, and $\tilde h$ are functions of $x\cdot\Delta$, $x\cdot\Delta'$, Δ^2, Δ'^2, and $\Delta\cdot\Delta'$. Following the standard procedure of first taking the Fourier transform of the functions and substituting Eqs. (4.3) to (4.7) into Eq. (4.2) we obtain after a straightforward, albeit tedious, calculation the following asymptotic expression for the structure functions:

$$W_1 \approx \sum_{J,J'} C_{J,J'}\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{J+J'} F_1(J,J')\,,$$

$$\begin{aligned} q\cdot\Delta W_2 &\approx \sum_{J,J'} C_{J,J'}\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{J+J'} F_2(J,J')\,, \\ q\cdot\Delta W_3 &\approx \sum_{J,J'} C_{J,J'}\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{J+J'-2} F_3(J,J')\,, \\ q\cdot\Delta W_4 &\approx \sum_{J,J'} C_{J,J'}\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{J+J'-1} F_4(J,J')\,,\end{aligned} \tag{4.8}$$

where $F_i(J,J')$ are functions of $\omega_R = Q^2/2q\cdot\Delta$ and Δ^2 only.

We assume that the indicated scaling behavior remains valid after the Sommerfeld-Watson transformation. Translated into H's our result states that

$$\begin{aligned} H^{++} &\approx -F_1\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}, \\ H^{00} &\approx \left[F_1 + \frac{1}{2\omega_R}(F_2+F_3+F_4)\right]\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}, \\ q\cdot\Delta H^{+-} &\approx -\tfrac{1}{2}\Delta^2 F_2\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha} \\ \sqrt{q\cdot\Delta}\, H^{+0} &\approx -\left(\frac{-\Delta^2}{\omega_R}\right)^{1/2}[F_2 + \tfrac{1}{2}F_4]\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha},\end{aligned} \tag{4.9}$$

with $F_i = F_i(\alpha, \omega_R, \Delta^2)$ and $\alpha = \alpha(\Delta^2)$. Referring to Eq. (2.7) we see that in the deep-inelastic Regge limit the differential cross section (a) becomes independent of azimuthal angle ϕ, (b) when multiplied by $(1-x)$ becomes linear in x. These two characteristic features should be relatively easy to observe.

It may be noted that the manner in which $q\cdot P$ appears in the combination $(q\cdot P/q\cdot\Delta)$ is familiar to workers of the triple-Regge region.[18]

The scaling functions F_i may be expressed in terms of (the Fourier transforms of) the bilocal functions appearing in Eq. (4.7),

$$\begin{aligned} F_1(J,J') &= C(-1)^{J+J'}\tfrac{1}{2}[f_{J,J'}^{(J+J')}(\omega_R) + g_{J,J'}^{(J+J'-1)}(\omega_R)]\,, \\ F_2(J,J') &= -C(-1)^{J+J'}[\omega_R f_{J,J'}^{(J+J')}(\omega_R) + (J+J')f_{J,J'}^{(J+J'-1)}(\omega_R)]\,, \\ F_3(J,J') &= C(-1)^{J+J'}(J+J'-1)g_{J,J'}^{(J+J'-2)}(\omega_R)\,, \\ F_4(J,J') &= C(-1)^{J+J'}[(J+J')f_{J,J'}^{(J+J'-1)}(\omega_R) - \omega_R g_{J,J'}^{(J+J'-1)}(\omega_R) - (J+J'-1)g_{J,J'}^{(J+J'-2)}(\omega_R)]\,,\end{aligned} \tag{4.10}$$

where the superscript on $f_{J,J'}^{(n)}$ denotes the nth derivatives with respect to ω_R. (For the special case of $J=J'=0$ we set $g_{0,0}^{(-2)}=g_{0,0}^{(-1)}=f_{0,0}^{(-1)}=0$ and $f_{0,0}^{(0)}=h_{0,0}$.) The fact that only two of the many bilocal functions in Eq. (4.7) contribute to the scaling functions F_i leads to two nontrivial relations among the F_i's, one of which is in fact independent of J and J' and does not involve differentiation with respect to ω_R. We simply assume that the Sommerfeld-Watson summation merely amounts to replacing $(J+J')$ by $2\alpha(\Delta^2)$. Thus we obtain two relations:

$$2\omega_R F_1+F_2+F_3+F_4=0\,, \tag{4.11}$$

$$\omega_R\frac{d^2F_3}{d\omega_R{}^2}+4\alpha\frac{dF_3}{d\omega_R}+(2\alpha-1)\left(\frac{dF_4}{d\omega_R}-4\alpha F_1\right)=0\,. \tag{4.12}$$

They are consequences of the tensor structure of the light-cone commutator Eq. (4.6) and supposed to be true for a finite range of small Δ^2.

The relation in Eq. (4.11) has a comfortable physical interpretation: It merely corresponds to the statement that H^{00} scales to zero in the deep-inelastic Regge limit. This has the observable consequence that the differential cross section in Eq. (2.7) becomes independent of x when multiplied by $(1-x)$.

The relation in Eq. (4.12) is more obscure. In the region of small ω_R, however, it may simplify somewhat. This corresponds to the so-called triple-Regge region $q\cdot P\gg q\cdot\Delta\gg Q^2$ where H^{ab} behaves like[19]

$$H^{ab}\sim\left(\frac{qP}{q\Delta}\right)^{2\alpha(\Delta^2)}(q\cdot\Delta)^{\alpha_P(0)-|a-b|}\,.$$

We suppose[20] that the leading trajectory is the Pomeranchukon α_P. In that case $F_1(\omega_R)\sim\omega_R{}^{-1}$ and $F_{2,3,4}(\omega_R)\sim\omega_R{}^0$ as $\omega_R\to 0$. From our experience with single-arm electroproduction we may expect the relevant range to be $0\lesssim\omega_R\lesssim 0.3$. In this range of ω_R, the relation in Eq. (4.12) becomes simply that $F_4\sim 0$, which in turn implies that H^{+-} and H^{+0} are related in this region.

V. DISCUSSION

Our analysis involves extrapolations through large distances. However, we are not interested in any detailed aspects of our van Hove analysis but only in the general behavior of the various structure functions. It may be reasonable to assume that our results have more general validity than the details of the analysis. To begin with, predicted scaling properties are such that, when Eq. (4.9) is substituted into the general relations among the helicity and invariant structure functions [Eq. (A21) (or, rather, its inverse)], no extra constraints result. This is certainly not true in general.[21] [In this sense, with some experience derived from works in the triple-Regge asymptotics, one could have "guessed" our result Eq. (4.9) from the relations in Eq. (A21).] Furthermore, our assumption is certainly valid if the conjecture of Ellis[2] concerning the light-cone structure of multilocal operators turns out to be correct. In fact this stronger assumption would lead to similar scaling behavior in a wider kinematic range[22] (i.e., the restriction of $q\cdot P\gg q\cdot\Delta$, Q^2 may be removed).

Of the two relations in Eq. (4.11) and (4.12) we expect that the first one is more credible. It comes from a relation independent of J and does not involve differentiation with respect to ω_R. We note that it is compatible with Eq. (4.9) and positivity conditions (Appendix). Another argument for its validity comes from the so-called inclusive sum rules.[23] They relate an integral of the coincidence cross sections over d^3p' to the single-arm cross sections. The zeroth-moment sum rule reads (Appendix)

$$(\nu^2+Q^2)^{1/2}n(Q^2,\nu)H^{ab}(Q^2,\nu)=\tfrac{1}{2}\pi\left(\frac{m'}{m}\right)\int dM_x{}^2d\Delta^2H^{ab}(Q^2,\nu,M_x{}^2,\Delta^2)\,, \tag{5.1}$$

where $(ab)=(++)$ or (00). $n(Q^2,\nu)$ denotes the multiplicity of the detected hadron. The inclusive "sum rule," being merely a statement on combinatorics, is devoid of any dynamical content. However, they do constrain various scaling behaviors (although in an essentially trivial manner). Strictly speaking, our result that $H^{00}(Q^2,\nu,M_x{}^2,\Delta^2)\approx 0$ in the limit $\nu\gg M_x{}^2$, Q^2 and Q^2 growing large with $Q^2/M_x{}^2$ and Δ^2 fixed does not follow immediately from positivity and the Callan-Gross relation, which states that $H^{00}(Q^2,\nu)\approx 0$ as $Q^2\to\infty$ with Q^2/ν fixed. In practice, however, $H^{00}(Q^2,\nu)$ is known to be small everywhere in the Q^2-ν plane. In particular, the assumption that $H^{00}(Q^2,\nu)\to 0$ as $\nu\to\infty$ with Q^2 fixed has been introduced[24] in the literature. In that case the validity of Eq. (4.11) will not be surprising.

The relation in Eq. (4.12) is on a rather different footing. While Eq. (4.11) expresses the fact that the leading scaling component in H^{00} is zero, Eq. (4.12) involves the nonleading $(q\cdot\Delta)H^{+-}$ and $(q\cdot\Delta)^{\frac{1}{2}}H^{+0}$ terms. As it turns out,[25] these functions are sensitive to "twist-four" operators[6] in a light-cone expansion. Consequently, whereas Eq. (4.12) is certainly true in a free-quark model, it is not likely that it can be maintained in, say, the quark-gluon model. In this way this curious relation provides us with a test of the light-cone expansion up to twist-four as seen in a free quark model. Experimentally this, of course, requires the more difficult measurements of H^{+-} and H^{+0} in the deep-inelastic limit.

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APPENDIX

(a) Variables. The relation between the set of variables used in the paper $\{\psi, \phi; \nu, Q^2, \Delta^2, M_x^2\}$ and the laboratory variables $\{\epsilon, \epsilon', E', \theta, \theta', \phi_L\}$ are (E' is the energy of the detected hadron and the angles θ, θ', and ϕ_L are defined in Fig. 5)

$$\nu = \epsilon - \epsilon', \tag{A1}$$

$$Q^2 = 2\epsilon\epsilon'(1-\cos\theta), \tag{A2}$$

$$\Delta^2 = m^2 + m'^2 - 2mE', \tag{A3}$$

$$M_x^2 = \Delta^2 - Q^2 + 2\nu(m - E') + 2(E'^2 - m'^2)^{1/2} \times[(\epsilon - \epsilon'\cos\theta)\cos\theta' - \epsilon'\sin\theta\sin\theta'\cos\phi_L]. \tag{A4}$$

The expressions for ψ and ϕ in the laboratory frame are already given in Eqs. (2.3) and (2.4). We note in particular that the azimuthal angle ϕ used in the paper is *not* the conventional laboratory azimuthal angle ϕ_L.[26] Comparing Eq. (2.4) to

$$\cos\phi_L = \frac{\vec{k}\times\vec{k}'}{|\vec{k}\times\vec{k}'|}\cdot\frac{\vec{k}\times\vec{p}'}{|\vec{k}\times\vec{p}'|}, \tag{A5}$$

we have

$$\cos\phi = [\epsilon'\sin\theta\cos\theta' + (\epsilon - \epsilon'\cos\theta)\sin\theta'\cos\theta_L]\cdot[(\epsilon-\epsilon'\cos\theta)^2\sin^2\theta' + \epsilon'^2\sin^2\theta(\cos^2\theta' + \sin^2\theta'\sin^2\phi_L) + 2\epsilon'(\epsilon - \epsilon'\cos\theta)\sin\theta\cos\theta'\sin\theta'\cos\phi_L]^{-1/2}. \tag{A6}$$

(b) Brick-wall frame and the O(2, 1) formalism. The process depicted in Fig. 1 is given as

$$T_{\lambda',\lambda} = e^2\bar{u}(k'\lambda')\gamma_\mu u(k\lambda)\frac{1}{q^2}\langle(\text{out})n, p'|J^\mu(0)|p\rangle, \tag{A7}$$

λ and λ' being the leptonic helicity labels. We shall evaluate it in the brick-wall (BW) frame defined by

$$q^\mu = \sqrt{Q^2}\,(0, 0, 0, 1). \tag{A8}$$

An arbitrary configuration in this BW frame is then described by the two parameters of the Lorentz transformations, which leaves q^μ invariant as in Eq. (A8). Explicitly, they consist of a boost along the x axis by the hyperbolic angle ψ, followed by a rotation around the z axis by an angle ϕ (Fig. 6).

These operations are defined with respect to a *special* BW frame in which $k_s^\mu = \frac{1}{2}\sqrt{Q^2}\,(1, 0, 0, 1)$.

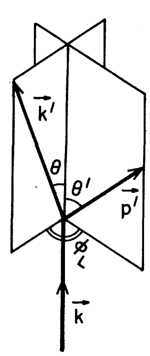

FIG. 5. Kinematics in the laboratory frame, z axis being defined by the incoming beam $\vec{k}$.

Therefore, when expanded on the basis

$$\epsilon^{(\pm)\mu} = (0, \mp 1, -i, 0)/\sqrt{2}, \quad \epsilon^{(0)\mu} = (1, 0, 0, 0), \tag{A9}$$

the leptonic vertex in this special frame is a function of Q^2 alone:

$$\bar{u}(k_s'\lambda')\gamma^\mu u(k_s\lambda) = \sqrt{2Q^2}\,(2\lambda)[\delta_{1,\lambda+\lambda'}\epsilon^{(+)\mu} - \delta_{-1,\lambda+\lambda'}\epsilon^{(-)\mu}]. \tag{A10}$$

The leptonic vertex in a general BW frame, in which $k^\mu = \frac{1}{2}\sqrt{Q^2}\,(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, 1)$, may be obtained easily from Eq. (A10) and the O(2, 1) transformation properties of $\{\epsilon^{(a)\mu}\}$:

$$\mathrm{O}^{-1}(\psi,\phi)\epsilon^{(a)\mu}\mathrm{O}(\psi,\phi) = D^a{}_b(\psi,\phi)\epsilon^{(b)\mu}, \tag{A11}$$

with

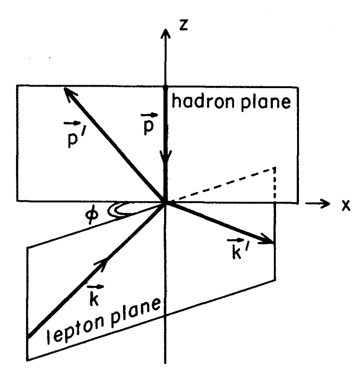

FIG. 6. Kinematics in the BW frame with $\vec{q}$ and $\vec{p}$ lying on z axis. (x-y plane is the "brick-wall.") ϕ is the azimuthal angle contained by the leptonic plane ($\vec{k}$, $\vec{k}'$) and the x-z hadronic plane ($\vec{p}$, $\vec{p}'$).

$$D^a{}_b(\psi,\phi)=\begin{pmatrix} e^{-i\phi}\frac{1}{2}(1+\cosh\psi) & -\frac{\sinh\psi}{\sqrt{2}} & e^{i\phi}\frac{1}{2}(1-\cosh\psi) \\ -e^{-i\phi}\frac{\sinh\psi}{\sqrt{2}} & \cosh\psi & e^{i\phi}\frac{\sinh\psi}{\sqrt{2}} \\ e^{-i\phi}\frac{1}{2}(1-\cosh\psi) & \frac{\sinh\psi}{\sqrt{2}} & e^{i\phi}\frac{1}{2}(1+\cosh\psi) \end{pmatrix}. \qquad \text{(A12)}$$

We are thus led immediately to a simple factorized form[12] for the scattering amplitude Eq. (A7),

$$T_{\lambda',\lambda}=\frac{e^2}{Q^2}\,j^{(a)}_{\lambda',\lambda}(Q^2)D_{ab}(\psi,\phi)J^{(b)}(Q^2,q\cdot P,q\cdot\Delta,\Delta^2)\,, \qquad \text{(A13)}$$

where the only nonzero leptonic vertices are $j^{(+)}_{1/2,\,1/2}=-j^{(-)}_{-1/2,\,-1/2}=(2Q^2)^{1/2}$. This expression for $T_{\lambda',\lambda}$ and Eq. (A12) show that the azimuthal angle ϕ plays a physically interesting role. We are instructed to consider the leptons as the source of a virtual photon of various polarization, and the amplitudes corresponding to the various polarizations differ only by appropriate powers of $e^{i\phi}$.

(c) Intensity distribution. For a longitudinally polarized (ρ) lepton beam the intensity distribution may be written as

$$d\sigma(\rho)\sim\tfrac{1}{2}(1+\rho)\,|T_{1/2,\,1/2}|^2+\tfrac{1}{2}(1-\rho)\,|T_{-1/2,\,-1/2}|^2 =I_u+\rho I_p\,. \qquad \text{(A14)}$$

Using Eqs. (A13) and (A12) and the definition for H^{ab} in (2.8) we have

$$\begin{aligned} I_u=&\tfrac{1}{2}(1+\cosh^2\psi)(H^{++}+H^{--})+\sinh^2\psi\, H^{00}\\ &-\frac{1}{\sqrt{2}}\sinh 2\psi[\cos\phi\,\mathrm{Re}(H^{+0}-H^{-0})\\ &\qquad -\sin\phi\,\mathrm{Im}(H^{+0}+H^{-0})]\\ &-\sinh^2\psi(\cos 2\phi\,\mathrm{Re}H^{+-}-\sin 2\phi\,\mathrm{Im}H^{+-})\,, \end{aligned} \qquad \text{(A15)}$$

$$\begin{aligned} I_p=&\cosh\psi(H^{++}-H^{--})\\ &-\sqrt{2}\,\sinh\psi[\cos\phi\,\mathrm{Re}(H^{+0}+H^{-0})\\ &\qquad -\sin\phi\,\mathrm{Im}(H^{+0}-H^{-0})]\,. \end{aligned} \qquad \text{(A16)}$$

So far the discussion is completely general. The above decomposition of intensity distribution with respect to the "trivial" variables ψ and ϕ should be valid for the general electroproduction process in which an arbitrary number of hadrons are detected. (In that case ϕ may be defined as the azimuthal angle of the final lepton with respect to some hadronic plane.) For process (1.1), in which only one hadron is detected in the final state, not all helicity form factors are independent. Parity invariance [i.e., viewing process (2.5) as a two-body to two-body reaction] leads to $H^{ab}=(-1)^{a-b}H^{-a\,-b}$. In this way we obtain the final expression in Eq. (2.7).

(d) Inclusive sum rules. The formalism developed in the last section can be readily applied to the single-arm process (1.2). Clearly the cross section is independent of ϕ and only H^{00} and $H^{++}(=H^{--})$ survive. They are, apart from a simple factor, just the familiar scalar and transverse cross sections:

$$\begin{aligned} \sigma_s(\nu,Q^2)&=2\pi^2\alpha\left(\frac{2m}{2m\nu-Q^2}\right)H^{00}(\nu,Q^2)\,,\\ \sigma_T(\nu,Q^2)&=2\pi^2\alpha\left(\frac{2m}{2m\nu-Q^2}\right)H^{--}(\nu,Q^2)\,. \end{aligned} \qquad \text{(A17)}$$

These structure functions are related to our H^{ab}'s of the process (1.1) by the so-called "inclusive sum rules"[23]:

$$(\nu^2+Q^2)^{1/2}n(\nu,Q^2)H^{ab}(\nu,Q^2) =\tfrac{1}{2}\pi\left(\frac{m'}{m}\right)\int dM_x{}^2d\Delta^2H^{ab}(\nu,Q^2,\Delta^2,M_x{}^2)\,, \qquad \text{(A18)}$$

where $(a,b)=(0,0)$ or $(+,+)$. $n(\nu,Q^2)$ is the multiplicity of the detected hadron.

(e) Positivity conditions. For any complex four-vector η^μ we have the condition $\eta^\mu W_{\mu\nu}\eta^{\nu *}\geq 0$. In terms of $\{H^{ab}\}$,

$$\begin{aligned} &H^{00}\geq 0\,,\\ &H^{++}\geq |H^{+-}|\geq 0\,,\\ &H^{00}(H^{++}-H^{+-})\geq 2\,|H^{+0}|^2\,. \end{aligned} \qquad \text{(A19)}$$

(f) Cross section of single-pion electroproduction. Our helicity structure functions $\{H^{ab}\}$ are essentially the cross sections of Hand often used in works on single-pion electroproduction.[1] Explicitly,

$$\begin{aligned} &H^{++}=c\sigma_U\,,\\ &H^{00}=c\sigma_L\,,\\ &H^{+-}=-c\sigma_T\,,\\ &\mathrm{Re}H^{+0}=-\tfrac{1}{2}c\sigma_I\,, \end{aligned} \qquad \text{(A20)}$$

with

$$c=\frac{4}{\alpha m}(2m\nu-Q^2)(\nu^2+Q^2)^{1/2}\delta(M_x{}^2-\mu_\pi{}^2)\,.$$

(g) Helicity and invariant structure functions. The $\{H^{ab}\}$ of Eq. (2.8) are related to $\{W_i\}$ of Eq. (2.9) as

$$\begin{aligned}
H^{++} &= H^{--} = -W_1 + \tfrac{1}{2}A^2(W_2+W_3-W_4),\\
H^{+-} &= H^{-+} = -\tfrac{1}{2}A^2(W_2+W_3-W_4),\\
H^{00} &= W_1 + B^2W_2 + C^2W_3 + BCW_4,\\
\mathrm{Re}H^{+0} &= \mathrm{Re}H^{0+} = -\mathrm{Re}H^{-0} = -\mathrm{Re}H^{0-}\\
&= \frac{A}{\sqrt{2}}\left[-BW_2 + CW_3 + \tfrac{1}{2}(B-C)W_4\right],\\
\mathrm{Im}H^{+0} &= -\mathrm{Im}H^{0+} = -\mathrm{Im}H^{-0} = \mathrm{Im}H^{0-}\\
&= \frac{A}{\sqrt{2}}\tfrac{1}{2}(B+C)W_5,
\end{aligned} \tag{A21}$$

where

$$A = \left(P\cdot\Delta - \frac{q\cdot Pq\cdot\Delta}{q^2} - \frac{N\cdot PN\cdot\Delta}{N^2}\right)^{1/2},$$

$$B = \frac{N\cdot\Delta}{(N^2)^{1/2}}, \quad C = \frac{N\cdot P}{(N^2)^{1/2}},$$

with

$$N_\mu = P_\mu - \frac{P\cdot q}{q^2}q_\mu. \tag{A22}$$

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[13]See, for example, A. Pais and S. B. Treiman, Ref. 3.

[14]A. Pais and S. B. Treiman, in *Problems of Theoretical Physics* (Nauka, Moscow, 1969), p. 257.

[15]As was mentioned, the intention is to apply our analysis to the case in which one detects the recoil proton. Some readers may be discomforted by the thought of visualizing the Pomeranchukon in the van Hove manner. Indeed, it is generally believed that no particle lies on the Pomeranchukon trajectory and that diffraction scattering is generated by the shadow of inelastic processes. Nevertheless, there are mounting evidences [R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. Letters 26, 1515 (1971); Phys. Rev. D 4, 3439 (1971)] supporting the hypothesis that the Pomeranchukon communicates with hadrons through the f and f' trajectories; if that is true, our analysis may be justifiably applied.

[16]The summation should only run over even (odd) J and J' if the trajectory under consideration has even (odd) signature. We absorb the appropriate signature factor $1\pm e^{i\pi j}$ into $G(J)$.

[17]The light-cone commutator in Eq. (4.6) may be rendered manifestly gauge-invariant by adding nonleading terms as discussed by Gross and Treiman (Ref. 6).

[18]See, for example, H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Letters 26, 937 (1971).

[19]We assume that the Toller quantum number of the Pomeranchukon $M=0$. Some evidence for this comes from $\gamma p\to\pi X$. H. D. I. Abarbanel and D. J. Gross, Phys. Rev. Letters 26, 732 (1971); Phys. Rev. D 5, 699 (1972).

[20]The possibility that the so-called triple-Pomeranchukon vertex vanishes (at $\Delta^2=0$) has been raised, Ref. 19.

[21]For example, the naive argument that H^{ab}, being of the same dimensionality, should scale similarly (see, for instance, Frishman *et al.*, Ref. 2) will lead to two constraints among the structure functions. We note that coefficients in Eq. (A19) have the following limit (3.1): $A\to(-\Delta^2)^{1/2}$, $B\to q\cdot\Delta/\sqrt{Q^2}$, $C\to q\cdot P/\sqrt{Q^2}$.

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[25]T. P. Cheng and A. Zee, Phys. Rev. D (to be published), where a more detailed discussion of this point will be presented, in particular for the case of deep-inelastic scattering on a spin-one target.

[26]In the literature the structure functions as defined by Drell and Yan (Ref. 2) are often used. We note that in defining $W_{1,2}^{\mathrm{DY}}$ these authors have averaged over ϕ_L (holding ν, Q^2, Δ^2, $M_x{}^2$ fixed) instead of over the "trivial" variable ϕ. Consquently, the relation between $W_{1,2}^{\mathrm{DY}}$ and the structure functions used in this paper is rather complicated.