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Heavy particle electroweak loop effects in extra-dimensional models with bulk neutrinos

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Abstract

One way to detect the presence of new particles in theories beyond the standard model is through their contribution to electroweak loop effects. We comment on the importance of a consistent inclusion of their mixing angles to ensure that the physical requirement of heavy particle decoupling is fulfilled. We illustrate our points by a detailed discussion of the lepton flavor changing effect $\mu \to e\gamma$, investigated recently by Kitano, in the Randall–Sundrum model. Our remarks are equally applicable to models with large compactified dimensions where bulk neutrinos are introduced to account for the observed neutrino oscillations. © 2001 Published by Elsevier Science B.V.

1. Introduction

There is considerable interest in the attempts of solving the gauge hierarchy problem in the context of theories invoking extra spatial dimensions. One category of models assumes that the extra dimensions are large. Mass hierarchy results from the large volume effect [1]. Another class makes use of a non-factorizable metric with a warp factor, leading to exponential suppressions of Planck scale masses for the relevant fields which are assumed to reside on the "visible" 3-brane [2]. Only gravitons and, in some models, also other fields, can propagate in the extra-dimensional space. Such a bulk field will have a tower of Kaluza–Klein states with ever increasing masses. These states often provide us with definitive signatures of these extra dimension theories. If such bulk fields can mix with or-

dinary SM fields, their presence can in principle be detected through their contributions to the electroweak loop effects. There exist already a substantial body of literature discussing the constraints on such KK states by using the existing, or future, electroweak precision data [3]. In this Letter we wish to emphasize the importance of a proper and complete inclusion of the mixing angle effects so that the physically sensible requirement of the decoupling theorem can be satisfied.

We believe that the study of any physical phenomena at a given distance scale should not depend sensitively on our knowledge of the physics on much shorter scales, heavy particles should decouple from low-energy processes. Namely, the effects of heavy particles in the virtual intermediate states are suppressed by inverse powers of the heavy particle masses [4]. This comes about because the relevant amplitudes are reduced by the heavy particle propagators. However, if the heavy mass comes from spontaneous symmetry breaking, the corresponding Yukawa coupling

is also large and this can neutralize the large mass of the denominator. This may lead to a violation of the decoupling theorem in the low energy effective theory [5]. For example, in the Standard Model, the ρ parameter grows with m_t^2 and is quite sensitive to the value of m_t. In fact, this is one of the clue to the t-quark mass before its discovery. On the other hand, if the large mass can be attributed to a gauge invariant mass term, then decoupling should be effective because here one does not need a large Yukawa coupling. Often in a model, particles have a mixture of bare and Yukawa-coupling-induced masses. These decoupling effects may show up as mass-suppressed mixing angles. Masses and mixing angles are often related because they all follow from the same (non-diagonal) mass matrices.

A prototypical case is the seesaw mechanism for generating neutrino masses [6]: besides the ordinary light neutrinos with masses $m_\nu \simeq \hat{m}^2/\widehat{M}$, there is also at least one other superheavy neutrino with a mass $m_N \simeq \widehat{M}$. (The small mass is assumed to have a magnitude comparable to the masses of ordinary quarks and charged leptons, $\hat{m} \simeq 1$ GeV, the large mass being an intermediate mass scale below the Planck scale, $\widehat{M} \simeq 10^{12}$ GeV.) In the charged weak current, the charged lepton is coupled to the combination $(\cos\theta|\nu\rangle + \sin\theta|N\rangle)$, where $|\nu\rangle$ stands for some superposition of the light neutrino states with mixing angles that are not necessarily small, but the angle θ is mass-suppressed

$$\theta \simeq \frac{\hat{m}}{\widehat{M}}. \tag{1}$$

Such a mixing angle simply reflects the property of the mass matrix that, in the $\widehat{M} \to \infty$ limit, the mixing of the singlet neutrino goes to zero. The presence of non-zero neutrino masses naturally leads to flavor violation loop effects such as $\mu \to e\gamma$. Both light and heavy neutrinos contribute, leading to a branching ratio [7]

$$B(\mu e\gamma) = \frac{3\alpha}{8\pi}\zeta^2\theta^4, \tag{2}$$

where α is the fine structure constant. The factor ζ being some product of the mixing coefficients among light neutrinos is not expected to be particularly small. Had one not taken into account of the fact that the heavy-light mixing angle θ is mass-suppressed, one would erroneously conclude that the heavy neutrino

did not decouple (and thus giving rise to an unacceptably large branching ratio). But in this representation of the neutrino states, decoupling manifest itself in the form $\theta^4 = (\hat{m}^2/\widehat{M}^2)^2 = m_\nu^2/m_N^2$ which yields an immeasurably small branching ratio — because of the superheavy neutrino mass in the denominator, as well as the tiny light neutrino mass in the numerator. The seesaw model of neutrino mass is considered to be an attractive possibility because the presence of such self-consistent features. We suggest that any physically sensible theory containing superheavy particles would have this type of properties that automatically ensures that the heavy states are decoupled in low energy processes.

The mass suppressed mixing angle follows from a special feature of the seesaw neutrino mass matrix — the absence of Majorana mass terms for the left-handed neutrino and a superheavy entry for the right-handed neutrino mass term. In essence, the reason that the decoupling holds in this case is due to the fact the large mass can be realized by having large bare mass (ν_R being a SM singlet) without having large Yukawa coupling. That mass matrices have the structure which gives rise to decoupling is rather common in models involving heavy particle states. Thus the question of mass suppressed mixing angles is very important in our consideration of heavy particle contribution to low energy loop effects. In this Letter we shall illustrate our points in the Randall–Sundrum model [2] with bulk neutrinos [8]. The investigation of lepton flavor violation loop effects in this context have recently been carried out by Kitano [9]. Here we complete his discussion, in particular with respect the possibility of extracting a meaningful bound on the heavy neutrino mass.

2. Mixing angles in the RS model with bulk neutrinos

The Randal–Sundrum model presupposes a five-dimensional spacetime. The extra spatial dimension is taken to be a compactified S^1/Z_2 orbifold with a coordinate $y = r_c\phi$, with r_c being the radius of the compact dimension and the angle ϕ having a range of $[-\pi, \pi]$ with opposite sides identified. There are two 3-branes fixed at $\phi = \pi$ (the "visible" brane containing the SM fields) and at $\phi = 0$ (the "hidden" brane, also

called the Planck brane). The resultant metric is non-factorizable:

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}\,dx^\mu\,dx^\nu - r_c^2\,d\phi^2, \tag{3}$$

where k, the bulk curvature, has the order of fundamental mass scale $\widehat{M}_5$ which is comparable to the Planck mass. The exponential warp factor $e^{-2kr_c|\phi|}$ causes a rescaling of the fields, which changes any mass parameter in the fundamental theory ($\simeq$ Planck scale) to an effective mass on the visible brane as $M = e^{-kr_c\pi}\widehat{M}_5$ ($\simeq$ electroweak scale $M_{\rm EW}$). Namely, for a choice of $kr_c \approx 12$, we can have

$$\epsilon = e^{-kr_c\pi} \approx 10^{-16}, \qquad M = \epsilon\widehat{M}_5 \approx 10^3 \text{ GeV}. \tag{4}$$

Because this mechanism does not allow any intermediate scale, between the Planck and electroweak, to appear, the seesaw mechanism for generating a naturally small neutrino mass is not applicable in the original RS model. In this connection, Grossman and Neubert [8] introduced a bulk fermion field.[1] They have shown that, for a reasonable range of parameters, the zero mode of such a fermion has a very small wavefunction at the physical brane and the Higgs generated mass can also be naturally suppressed. In this way, neutrino masses that are many orders smaller than $M_{\rm EW}$ can be obtained.

The bulk fermion (with mass M_b) has the Kaluza–Klein decomposition of

$$\Psi_5^{L,R}(x,\phi) = \frac{e^{2kr_c|\phi|}}{\sqrt{r_c}}\sum_n \hat{f}_n^{L,R}(\phi)\psi_n^{L,R}(x), \tag{5}$$

where the superscripts (L,R) signify the chirality states $\Psi_5^{L,R} = \frac{1}{2}(1 \mp \gamma_5)\Psi_5$, and $\{\hat{f}_n^{L,R}(\phi)\}$ are the appropriate sets of complete orthonormal functions (in this case some combinations of Bessel functions) normalized so that $\psi_n(x)$ has the canonical scale in four dimensions,

$$S_\psi = \int d^4x \left\{ \bar\psi_n(x)i\partial\!\!\!/\psi_n(x) - M_n\bar\psi_n(x)\psi_n(x) \right\}. \tag{6}$$

The KK fields $\psi_{n\neq0}^{L,R}(x)$ has electroweak scale masses $M_n = \epsilon k x_n$ with x_n (corresponding to zeros of some

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[1] Cancellation of parity anomaly requires that there be even number of bulk fermions. Since the presence of multiple bulk fermions should not introduce qualitative changes in our result, we shall ignore such complication and stick with one bulk fermion.

combinations of the Bessel functions) being of order one. The presence of such states brings hope for experimental searches, or equivalently, for severe constraints by known phenomenology. Our focus in this Letter is the proper accounting, in such analyses, of the important effects due to the mixing angles between these heavy states and the SM fields.

Grossman and Neubert [8] have shown that bulk fermion zero modes ($x_0 = 0$) exist. If we impose the orbifold symmetry $\phi \to -\phi$, then only one of the chiral zero modes survives. Let it be $\psi_0^R(x)$, which has a suppressed wavefunction on the visible brane

$$\hat{f}_0^R(\phi = \pi) \simeq \sqrt{\epsilon kr_c}\,\epsilon^{\nu-\frac{1}{2}} = {\rm O}(\epsilon^\nu)$$
$$\text{with } \nu = \frac{M_b}{k} > \frac{1}{2}. \tag{7}$$

where M_b is the bulk fermion mass parameter in the original 5-dimensional Lagrangian. Similarly, orbifold symmetry requires the wavefunctions for the left-handed KK excitations, when evaluated on the visible brane, to vanish $\hat{f}_n^L(\phi = \pi) = 0$ while those for the right-handed fields have values

$$\hat{f}_{n\neq0}^R(\phi = \pi) \simeq \sqrt{\epsilon kr_c} = {\rm O}(\epsilon^{1/2}). \tag{8}$$

Thus, $(\hat{f}_0^R/\hat{f}_{n\neq0}^R)_{\phi=\pi} = {\rm O}(\epsilon^{\nu-1/2})$ is quite small since ϵ is tiny and $\nu > 1/2$.

Relevant to our discussion of neutrino mass matrix, we shall only display the Yukawa interaction between the SM left-handed lepton doublet $L^L = (l^L, \nu_l^L)$, the right-handed bulk fermion Ψ_5^R, and the Higgs doublet $H = (h^+, h^0)$, with its conjugate being $\widetilde{H} = i\sigma_2 H^*$. Again for simplicity we shall suppress the lepton generation indices (e, μ, τ) at this stage,

$$S_Y = -\int d^4x\,\epsilon^4\widehat{Y}_5\left\{\bar{L}_5^L(x)\widetilde{H}_5(x)\Psi_5^R(x,\pi) + \text{h.c.}\right\}, \tag{9}$$

where the factor ϵ^4 originates from the square root of the metric determinant, and $\widehat{Y}_5$, the fundamental Yukawa coupling, is dimensionful, expected to be somewhat less than $\widehat{M}_5^{-1/2}$; and the fundamental fields $L_5^L(x)$ and $\widetilde{H}_5(x)$ can be replaced by the effective fields (which have the canonical normalizations in the four-dimensional spacetime): $\epsilon^{-3/2}L(x)$ and $\epsilon^{-1}H(x)$, respectively. After substituting in the KK decomposition of Eq. (5), the Yukawa interaction

in Eq. (9) has now the form:

$$S_Y = -\int d^4x \left\{ y_0 \bar{L}^L(x)\widetilde{H}(x)\psi_0^R(x) \right.$$
$$\left. + \sum_{n=1} y_n \bar{L}^L(x)\widetilde{H}(x)\psi_n^R(x) + \text{h.c.} \right\},$$
(10)

where $y_n = \widehat{Y}_5 \hat{f}_n^R(\phi = \pi)/\sqrt{\epsilon r_c}$. After using the estimates of Eqs. (7) and (8), and $\sqrt{k}\widehat{Y}_5 \lesssim 1$, the four-dimensional Yukawa couplings have the size of

$$y_0 \lesssim \epsilon^{\nu-\frac{1}{2}} \quad \text{and} \quad y_n \lesssim 1. \tag{11}$$

Spontaneous symmetry breaking due to the Higgs mechanism results in a non-zero vacuum expectation value for the neutral scalar field $\langle h^0 \rangle = v$ of the electroweak scale. Eq. (10) leads to mass terms:

$$m_0 \bar{\nu}_l^L \psi_0^R + \sum_{n=1} m_n \bar{\nu}_l^L \psi_n^R, \tag{12}$$

with the "Yukawa masses"

$$m_0 = y_0 v \lesssim \epsilon^{\nu-\frac{1}{2}} v \ll M_{\text{EW}},$$
$$\text{and} \quad m_n = y_n v \lesssim M_{\text{EW}}. \tag{13}$$

Combining with the Dirac mass terms of the KK states $\sum_{n=1} M_n \bar{\psi}^L \psi_n^R$, these mass terms can be written in a matrix form

$$\left(\bar{\nu}'^L, \ \bar{\psi}_1^L, \ \bar{\psi}_2^L, \ldots \right) \begin{pmatrix} m_0 & m_1 & m_2 & \cdots \\ 0 & M_1 & 0 & \cdots \\ 0 & 0 & M_2 & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$
$$\times \begin{pmatrix} \psi_0^R \\ \psi_1^R \\ \psi_2^R \\ \vdots \end{pmatrix}. \tag{14}$$

For simplicity, let us concentrate on the simplest nontrivial case by cutting off the $n > 1$ excitations, thus a neutrino mass term of $\overline{\Psi}'_L \mathbb{M} \Psi'_R$, where

$$\overline{\Psi}'_L = (\bar{\nu}_l^L, \ \bar{\psi}_1^L), \qquad \mathbb{M} = \begin{pmatrix} m_0 & m_1 \\ 0 & M_1 \end{pmatrix},$$
$$\Psi'_R = \begin{pmatrix} \psi_0^R \\ \psi_1^R \end{pmatrix}, \tag{15}$$

with $m_0 \ll m_1 \lesssim M_1$. The mass matrix can be diagonalized in terms of the mass eigenstates (ν, N) by

unitary transformation matrices $\mathbb{U}(\theta_L)$ and $\mathbb{V}(\theta_R)$ acting on the left- and right-handed fields, respectively:

$$\mathbb{U} \begin{pmatrix} \nu \\ N \end{pmatrix}_L = \begin{pmatrix} \nu_l^L \\ \psi_1^L \end{pmatrix} \simeq \begin{pmatrix} \nu_L + \theta_L N_L \\ -\theta_L \nu_L + N_L \end{pmatrix},$$
$$\mathbb{V} \begin{pmatrix} \nu \\ N \end{pmatrix}_R = \begin{pmatrix} \psi_0^R \\ \psi_1^R \end{pmatrix} \simeq \begin{pmatrix} \nu_R + \theta_R N_R \\ -\theta_R \nu_R + N_R \end{pmatrix}, \tag{16}$$

so that

$$\mathbb{U} \mathbb{M} \mathbb{V}^\dagger = \mathbb{M}_{\text{diag}} = \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix}, \tag{17}$$

with $m_\nu \simeq m_0$ being very small and $m_N \simeq M_1$ very large. The mixing angle for the left-handed field θ_L should be fairly small, while θ_R for the right-handed fields is even more suppressed:

$$\theta_L \simeq \frac{m_1}{M_1} \lesssim 1 \quad \text{and} \quad \theta_R \simeq \frac{m_0 m_1}{M_1^2} \lesssim \text{O}(\epsilon). \tag{18}$$

Next we will examine in some detail how such mixing angles will figure in the constraint that the electroweak loop effects, such as $\mu \to e\gamma$, will place on the new physics. Obviously for this purpose, we must have at least two distinctive lepton flavors: $\nu_l = \nu_e, \ \nu_\mu$. Thus the m_0 factor in (15) is now a 2×2 non-diagonal mass matrix, whose elements are of same order magnitude as before. The gauge and mass eigenstates in (15) and (16) must be expanded minimally to sets of three states:

$$\mathbb{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = \begin{pmatrix} \nu_e^L \\ \nu_\mu^L \\ \psi_1^L \end{pmatrix}. \tag{19}$$

The mass eigenstates $\{\nu_i\}$ correspond to two light neutrinos with masses $m_{\nu 1}$ and $m_{\nu 2}$, on the order of zero-mode Yukawa mass m_0, and one heavy neutrino with $m_{\nu 3} \simeq M_1$. (We have changed the label for the heavy neutrino from N to ν_3.) For simplicity, we shall assume that the unitary matrix $\mathbb{U}$ can be parametrized by two mixing angles: one being the rotation angle ω in the $(1, 2)$-plane, and the other being θ_L, the $(2, 3)$ light-heavy rotation angle

$$\mathbb{U}_{li} = \begin{pmatrix} \cos\omega & -\cos\theta_L \sin\omega & \sin\theta_L \sin\omega \\ \sin\omega & \cos\theta_L \cos\omega & -\sin\theta_L \cos\omega \\ 0 & \sin\theta_L & \cos\theta_L \end{pmatrix}. \tag{20}$$

We now proceed to the discussion of the $\mu e\gamma$ loop effects.

3. $\mu \to e\gamma$: heavy particle in the gauge boson loop

The decay amplitude for the $\mu(p) \to e(p-q) + \gamma(q)$ can be written as

$$
T(\mu e\gamma) = \frac{ie}{16\pi} \varepsilon_\lambda^*(q) \bar{u}_e(p-q) \sigma^{\lambda\rho} q_\rho
$$
$$
\times \left[A_+(1+\gamma_5) + A_-(1-\gamma_5) \right] u_\mu(p). \tag{21}
$$

The branching ratio (with $m_e = 0$) can then be expressed in terms of the invariant amplitudes $A_\pm$ as

$$
B(\mu e\gamma) = \frac{6e^2 M_W^4}{g^4 m_\mu^2} \left(|A_+|^2 + |A_-|^2 \right), \tag{22}
$$

where g is the weak gauge coupling, M_W the weak gauge boson mass.

First we discuss the invariant amplitudes $A_\pm^W$ coming from the gauge boson loop contribution $\mu^- \to (\nu_i W_\gamma^-) \to e^-$ where the photon is emitted by the charged W boson in the loop (as denoted by the subscript γ). The gauge boson coupling to the charged lepton and massive neutrinos is

$$
\mathcal{L}(Wl\nu_i) = \frac{g}{\sqrt{2}} \mathbb{U}_{li} \bar{l} \gamma^\alpha \frac{1}{2}(1-\gamma_5) \nu_i W_\alpha^- + \text{h.c.}, \tag{23}
$$

leading (after a detailed calculation [7]) to the amplitudes of $A_-^W = 0$ and

$$
A_+^W = \frac{g^2 m_\mu}{8\pi M_W^2} \sum_{i=1}^3 \mathbb{U}_{\mu i}^* \mathbb{U}_{ei} F\left(\frac{m_i^2}{M_W^2} \right), \tag{24}
$$

where the function

$$
F(z) = \frac{1}{6(1-z)^4} \big(10 - 43z + 78z^2 - 49z^3
$$
$$
- 18z^3 \ln z + 4z^4 \big), \tag{25}
$$

has limits of $F(0) = 5/3$ and $F(\infty) = 2/3$, respectively. In our case we have two light neutrinos $\nu_{1,2}$ (thus $z_{1,2} \simeq 0$) and one heavy one $z_3 \gg 1$, resulting in a branching ratio of

$$
B(\mu e\gamma)_W = \frac{3\alpha}{8\pi} \left| \sum_{i=1}^3 \mathbb{U}_{\mu i}^* \mathbb{U}_{ei} F\left(\frac{m_i^2}{M_W^2} \right) \right|^2
$$
$$
= \frac{3\alpha}{8\pi} \left| \mathbb{U}_{\mu 3}^* \mathbb{U}_{e3} \left[F(\infty) - F(0) \right] \right|^2
$$
$$
= \frac{3\alpha}{8\pi} \zeta^2 \theta_L^4, \tag{26}
$$

where $\zeta = \frac{1}{2}\sin 2\omega$, as seen in Eq. (20). We have used the orthogonality condition of the mixing matrix $\mathbb{U}$ when going to the second line. Allowing for a large μe mixing angle ω, the experimental limit [10] of $B(\mu e\gamma) \simeq 10^{-11}$ requires a heavy-light angle $\theta_L \simeq m_1/M_1 = O(10^{-2})$, which is small, but still plausible as we expect the Yukawa masses m_1 to be quite bit less than the Dirac (bare) mass M_1. Thus the measured value begins to give meaningful constraint on the model parameters. Our point is that the significant restriction is on the mixing angle, rather than on the KK masses directly. Note that if we had not taken into account the suppression due to the mixing angles we would get an unacceptable large $B(\mu \to e\gamma)$. Also in this case the large mass comes from bare mass and not from the large Yukawa coupling and we expect the decoupling to be valid [5]. Indeed $B(\mu \to e\gamma)$ vanishes in the limit $M_1 \to \infty$, after the behavior of the mixing angles is included.

4. $\mu \to e\gamma$: heavy particle in the scalar boson loop

In the minimal SM with massive neutrinos, the leading $\mu e\gamma$ amplitude comes from the gauge boson loop as discussed in the previous section. However, for models having more scalars beyond the one Higgs doublet, there could in principle be significant scalar boson loop contribution as well. Even for the minimal SM, it is instructive to consider the scalar boson case separately because the longitudinal gauge boson is simply the (unphysical) Higgs scalar boson. Being proportional to the fermion mass, such Yukawa coupling is the source of the decoupling violation — through the cancellation of the large mass in fermion propagator by the large Yukawa couplings.

The Yukawa interactions of the scalar boson ϕ to a charged lepton l and a massive neutrino ν_i can be parametrized by the chiral couplings $y_{li}^{(\pm)}$:

$$
\mathcal{L}(\phi l\nu_i) = \bar{l} \left[y_{li}^{(+)}(1+\gamma_5) + y_{li}^{(-)}(1-\gamma_5) \right] \nu_i \phi^- + \text{h.c.} \tag{27}
$$

We have performed a detailed calculation of the scalar boson loop amplitude, $\mu^- \to (\nu_i \phi_\gamma^-) \to e^-$, where the photon is emitted by the charged ϕ boson in the intermediate state, and found that, for heavy neutrino

intermediate state ($m_i \gg M_\phi$),

$$
\begin{aligned}
&A_+^\phi(\nu_{\text{heavy}}) \\
&= \frac{1}{\pi m_i^2}\left(\frac{m_\mu}{3} y_{ei}^{(+)*} y_{\mu i}^{(-)} + m_i y_{ei}^{(+)*} y_{\mu i}^{(+)}\right),
\end{aligned}
\tag{28}
$$

and, for light neutrino intermediates state ($m_i \ll M_\phi$),

$$
\begin{aligned}
&A_+^\phi(\nu_{\text{light}}) \\
&= \frac{1}{\pi M_\phi^2}\left(\frac{m_\mu}{6} y_{ei}^{(+)*} y_{\mu i}^{(-)} + m_i y_{ei}^{(+)*} y_{\mu i}^{(+)}\right).
\end{aligned}
\tag{29}
$$

The other chiral amplitudes A_-^ϕ have similar structures.

Let us first consider the SM case when the scalar is the would-be-Goldstone boson, and becomes the longitudinal gauge boson after spontaneous symmetry breaking. In the renormalizable R_ξ gauge, there is a scalar particle with a mass $M_\phi = \xi M_W$. The SM fermions obtain their masses through their couplings to the Higgs field, hence the Yukawa couplings are proportional to the fermion masses. If the VEV is written in terms of g and M_W, we have the explicit form of

$$
y_{ei}^{(+)*} = \frac{g m_i}{2\sqrt{2}\,M_W} U_{ei}^*, \qquad y_{ei}^{(-)*} = \frac{-g m_e}{2\sqrt{2}\,M_W} U_{ei}^*,
$$

$$
y_{\mu i}^{(+)} = \frac{g m_\mu}{2\sqrt{2} M_W} U_{\mu i}, \qquad y_{\mu i}^{(-)} = \frac{-g m_i}{2\sqrt{2}\,M_W} U_{\mu i}.
\tag{30}
$$

Substituting these relations into Eq. (28), we can check that heavy neutrino (ν_3) contribution is given by

$$
A_+^\phi(\nu_{\text{heavy}}) = \frac{g^2 m_\mu}{8\pi M_W^2} \mathbb{U}_{\mu3}^* \mathbb{U}_{e3}(2/3),
\tag{31}
$$

in agreement with the result in Eq. (24) with $F(\infty) = 2/3$. This shows that the heavy particle non-decoupling contribution to the $\mu e \gamma$ amplitude comes entirely from the Higgs boson loop [5].

For models with non-minimal Higgs structure, we have physical scalar particles with couplings that do not have a simple fermion mass dependence — in fact they are as a rule highly model-dependent. The result of (28) can then be translated, with $m_\mu/m_i \simeq 0$, into the branching ratio of

$$
B(\mu e \gamma)_{\phi \nu_i} = \frac{24\alpha}{\pi g^4}\left(\frac{M_W^4}{m_\mu^2 m_i^2}\right)\left(y_{ei}^{(+)*} y_{\mu i}^{(+)}\right)^2.
\tag{32}
$$

Clearly a naive assumption of $(y_{ei}^{(+)*} y_{\mu i}^{(+)}) = \text{O}(1)$ would lead to a meaninglessly weak bound on the heavy neutrino mass [2] of $m_i > 10^7$ TeV. Since the generalized Yukawa couplings $y_{li}^{(\pm)}$ do include small mixing angles, it seems more sensible to use the experimental result [10] of $B(\mu e \gamma) \lesssim 1.2 \times 10^{-11}$ to set a limit on the coupling and mixing angle combination:

$$
\left(y_{ei}^{(+)*} y_{\mu i}^{(+)}\right) \lesssim \frac{10^{-7}}{m_i \text{ (TeV)}},
\tag{33}
$$

where we have used the value of Fermi constant $G_F = \sqrt{2}\,g^2/8M_W^2 \simeq 10^{-5}/M_N^2$.

5. Discussion

We have focused on the Randall–Sundrum version of the extra-dimensional theory. However, our discussion is equally applicable to the original version where the suppression of the bulk field effects (gravitons, singlet-neutrinos, etc.) comes through the large volume of the extra-dimensional space. This is the case because the structure of the neutrino mass matrix is very similar in both versions of the theory [11].

In this Letter, we have concentrated on a single bulk neutrino. In principle, there is a whole tower of Kaluza–Klein states. Many authors [12] have attempted to sum over the contribution by the whole tower. We have not done so because we do not wish to confuse the issue of decoupling of a single heavy particle with the separate problem of how the sum of this infinite tower should behave. The individual heavy particle contribution is controlled by the heavy-light mixing angle, which is mass-suppressed. If one sums over an infinite number of such small terms, a "non-decoupling" result can be obtained. Clearly, this approach touches upon the difficult issue of convergence of the KK sum, with implications related to the possible presence of new physics at higher scales. Such

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[2] In this respect, we differ from the conclusion drawn in Ref. [9] where the scalar contribution to the branching ratio has been estimated to have the mass dependence of $(M_W/m_i)^4$, as compared to our result of Eq. (32). The author was also silent with regard to the implication of the apparent decoupling violation by the W-loop contribution, as stated in Eq. (24).

problems are quite different from the matter of single particle decoupling, which is the focus of this Letter.

Acknowledgements

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