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Scaling Behavior of Helicity Structure Functions

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We show that the general behavior $|q^2|^{|a-b|/2}\,W^{ab}(q^2,\,\omega)\sim F^{ab}(\omega)$ holds in the scaling region. W^{ab} is the absorptive forward Compton amplitude with virtual-photon helicity indices a, b. This leads to readily observable consequences in the azimuthal angular distributions of the semileptonic processes $l+p\to l'+p_1+\cdots+p_n+X$ and $l+l'\to p_1+\cdots+p_n+X$, where p_i are the detected hadrons, and in deep-inelastic scattering of leptons on polarized targets which includes polarized nucleon, virtual photon, and Reggeon. A study of the tensor properties of the light-cone expansion shows that, due to competition of certain higher-twist contributions, relations involving F^{ab} for $a\neq b$ are more model-dependent than relations involving only F^{aa}. Ambiguities which arise in gauge completions of light-cone expansion are illustrated by certain scaling-function sum rules.

I. INTRODUCTION

Consider the absorptive part of a general forward Compton amplitude (Fig. 1),

$$W^{ab}(q^2,\,\omega)=\int d^4x\,e^{iqx}\langle P(\beta)|[J^\mu(x),J^\nu(0)]|P(\alpha)\rangle \times\epsilon_\mu^{(a)}\epsilon_\nu^{(b)*}, \qquad (1.1)$$

where J_μ is the hadronic current. The initial and final states may have different helicity labels (they are, of course, of identical momentum configurations). The virtual photon has momentum q; its initial and final helicities are a and b ($a,b=0,\pm1$). In general the scattering state in Eq. (1.1) may be a multiparticle state with momenta $p_1,\ldots,p_n$. For the sake of notational simplicity we denote it by a momentum label P and helicities α and β. Similarly the variable ω generically denotes the set $\{\omega_i=-q^2/2qp_i\}$. Furthermore, we do not exhibit on the left-hand side of Eq. (1.1) information not essential for our considerations (e.g., the dependence of W^{ab} on α, β and on p_ip_j). In the simplest electroproduction process (for example the MIT-SLAC single-arm unpolarized target experiment) one only measures W^{ab} corresponding to Eq. (1.1) with a single (spin-averaged) nucleon state and $a=b$. In this paper we shall, however, concentrate on the scaling properties of W^{ab} for $a\neq b$. Such amplitudes shall be referred to as off-diagonal structure functions.

We now state our main result: In the Bjorken scaling limit[1] of $q^2\to-\infty$ with ω fixed (more explicitly, we take the generalized Bjorken limit of $q^2\to-\infty$ with $\{\omega_i\}$ and $\{p_ip_j\}$ fixed), W^{ab} behave according to the scaling law

$$W^{ab}(q^2,\,\omega)\sim|q^2|^{-|a-b|/2}F^{ab}(\omega). \qquad (1.2)$$

The off-diagonal structure functions are in general accessible to measurements in the following two types [(A) and (B)] of semileptonic processes.

(A) $l+p_1\to l'+p_2+\cdots+p_n+X,\quad (n\geq 2)$

where $p_2\cdots p_n$ denote the $(n-1)$ observed hadrons [Fig. 2(a)]. Generally the target is unpolarized and no final polarization is observed. (In this category we can also add the crossed (colliding-beam) reaction [Fig. 2(b)]: $l+\bar{l}'\to\bar{p}_1+p_2+\cdots+p_n+X$ ($n\geq 2$) to which all our remarks may be applied.)

(B) $l+p\to l'+X$,

where p is a polarized target. In (A) and (B) X denotes the missing hadron complex, l and l' are the initial and final leptons. Although in this paper we shall work explicitly only with electroproduction, it will be evident that our result (1.2) applies, without modification, to weak production processes as well. We now discuss the kinematical features of these two types of processes separately.

(A) In the one-photon-exchange approximation, process (A) has the general azimuthal angular distribution

$$I=I_1+I_2\cos\phi+I_3\sin\phi+I_4\cos2\phi+I_5\sin2\phi\,. \qquad (1.3)$$

In the laboratory the angle ϕ is defined by

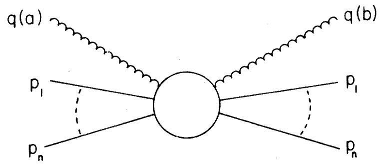

FIG. 1. Generalized forward Compton amplitude with initial and final photons in helicity states a and b.

$$\cos\phi = \frac{\vec{1}\times\vec{1}'}{|\vec{1}\times\vec{1}'|}\cdot\frac{(\vec{1}-\vec{1}')\times\vec{p}'}{|(\vec{1}-\vec{1}')\times\vec{p}'|}, \tag{1.4}$$

where $\vec{p}'$ is any momentum formed out of the observed hadrons. (We note that ϕ should not be confused with the conventional laboratory azimuthal angle contained between planes spanned by $\vec{1}$ with $\vec{1}'$ and $\vec{p}$, respectively. It represents a generalization of the Treiman-Yang angle and has been mentioned by a number of authors.[2]) For $n=2$ (i.e., single coincidence electroproduction[3]) parity invariance implies that $I_5=0$, and further, if the lepton beam is unpolarized, that $I_3=0$. It is straightforward to show that the coefficients of the $\cos(|a-b|\phi)$ and $\sin(|a-b|\phi)$ terms in (1.3) are directly proportional to semiconnected parts of W^{ab} as shown in (1.1). (For a discussion of coincidence electroproduction kinematics, see Ref. 3.) A number of authors[4] have pointed out that the scaling behavior of these structure functions is expected to be the same as the *full* absorptive amplitudes W^{ab}. (A more precise statement on this question will be given in Sec. III.) Consequently, our result (1.2) implies that ϕ-dependent terms scale away in the Bjorken limit as

$$\frac{I_{2,3}}{I_1}\sim|q^2|^{-1/2}, \quad \frac{I_{4,5}}{I_1}\sim|q^2|^{-1}. \tag{1.5}$$

This feature should be relatively easy to observe: One does not need to identify the detected final hadrons and one may also integrate over all other variables as long as the basic requirement of large q^2 and $\{qp_i\}$ is satisfied.

Comment: Abarbanel and Gross[5] have derived the behavior

$$W^{ab}(q^2,\{\nu_i\},\{p_ip_j\})\sim\beta^{ab}(q^2,\{\nu_i/\nu_1\},\{p_ip_j\}) \times\nu_1^{\alpha-|a-b|} \quad (i,j=1,\ldots,n) \tag{1.6}$$

as $\nu_1=qp_1\to\infty$ with ν_i/ν_1 and q^2 fixed, in other words, in the target-fragmentation region. Their derivation is based on Mueller's picture of inclusive reactions, factorization, and the assumed $M=0$ nature of the leading trajectory α. (M is the so-called Toller quantum number.[6]) We note that our result Eq. (1.2) is *not* necessarily implied by Eq. (1.6). Rather, they are complementary in the sense that if the scaling and Regge limits may be interchanged,[7] Eq. (1.6) dictates the small ω_1 behavior of $F^{ab}(\omega)$ in Eq. (1.2) with ω_i/ω_1 fixed, namely, that[8]

$$F^{ab}(\omega)\to(\omega_1)^{-\alpha+|a-b|}f^{ab}(\{\omega_i/\omega_1\},\{p_ip_j\}), \tag{1.7}$$

where f^{ab} is an unknown function. In other words, Eqs. (1.2) and (1.6) jointly imply that as $|q^2|\to\infty$,

$$\beta^{ab}(q^2,\{\nu_i/\nu_1\},\{p_ip_j\}) \sim|q^2|^{-\alpha+|a-b|/2}\gamma^{ab}(\{\nu_i/\nu_1\},\{p_ip_j\}), \tag{1.8}$$

where γ^{ab} is an unknown function.

(B) It is obvious that the off-diagonal structure functions are also accessible in the total inclusive electroproduction ($n=1$) when the target is polarized. W^{ab} with $a\neq b$ is just the usual (two-body) spin-flip forward Compton amplitude. In practice one may consider deep-inelastic scattering on (i) a polarized nucleon, (ii) an off-shell photon [Fig. 3(a)], and (iii) a Reggeon [Fig. 3(b)]. Case (i) has already received a good deal of attention in the literature.[9-12] The two spin-dependent structure functions νG_1 and $\nu^2 G_2$ are expected to scale (see Appendix A). Case (ii) can be extracted in a colliding-lepton-beam experiment [Fig. 3(a)]. A number of authors[13] have already considered this limit with the spin of the "target photon"

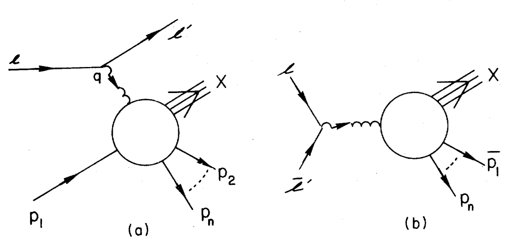

FIG. 2. Process (A): (a) Coincidence electroproduction; (b) colliding-lepton-beam production.

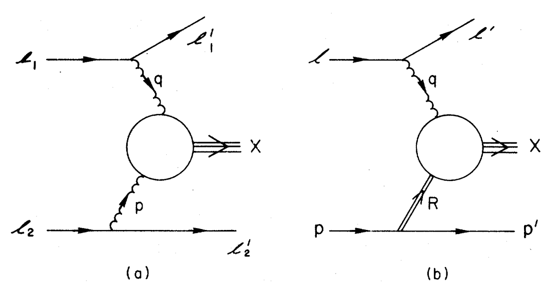

FIG. 3. Process (B): (a) "Polarized-photon target" as furnished by colliding-beam machine; (b) "polarized-Reggeon target" R as provided by coincidence electroproduction.

averaged (hence $a=b$). Since in such processes the photon in effect comes to us polarized without cost, we present a detailed discussion of deep-inelastic scattering on a polarized spin-1 target in Sec. IV. Case (iii) may be thought of as lepton scattering on a target of continuous spin. It was the subject of a recent study.[3] All these results illustrate the general kinematical theorem in Eq. (1.2).

For want of a better phrase we will refer to the behavior in Eq. (1.2) as a "kinematical suppression" of W^{ab} for $a\neq b$. This is in contradistinction to what we call "dynamical suppressions," which cause $F^{ab}(\omega)$ to vanish for certain helicity components a and b. The best-known example is provided by the Callan-Gross relation[14] which states that, depending upon the spin structure of the underlying dynamics, either $F^{00}(\omega)$ or $F^{++}(\omega)$ vanishes. As we will see in Sec. V and Appendix A, light-cone relations involving kinematically suppressed structure functions are in general less credible than relations not involving them. In particular, we show explicitly in Appendix A that the validity of certain sum rules depends on how gauge invariance on the light cone is implemented.

The familiar single-arm situation in which the scaling of W_1 and νW_2 leads to the scaling of W^{++} and W^{00} has fostered the erroneous impression that the scaling behavior of structure functions may be deduced from dimensional arguments alone.[15] Our result shows this to be a falsehood.[16]

We will give two derivations of our results Eq. (1.2). The first derivation, to be given in Sec. II, is less general and applies only to process (B). It rests on O(4) symmetry and specific assumptions about the Bjorken-Johnson-Low (BJL) limit.[17] The second derivation involves the light-cone behavior of the product of two currents and constitutes Secs. III and V.

II. DISPERSION RELATION AND BJORKEN-JOHNSON-LOW LIMIT OF FORWARD COMPTON AMPLITUDE

Consider the off-shell forward Compton amplitude

$$T^{ab}(\nu, q^2)=i\int d^4x\, e^{iqx}\langle p(d)|\, T(J^\mu(x)J^\nu(0))\,|p(c)\rangle \times\epsilon_\mu^{(b)*}\epsilon_\nu^{(a)}\,. \qquad (2.1)$$

On the left-hand side $\nu=p\cdot q$ and the helicity indices of target c and d have been suppressed. (Angular momentum conservation implies that $a-c=b-d$.) Thus in general T^{ab} stands for a number of amplitudes depending on the target spin. Our convention for the photon polarization vectors is as follows: For $q^\mu=(0,0,0,q^3)$ and $q^3>0$,

$$\epsilon^{(\pm)}(q)=-(0,\pm1,i,0)/\sqrt{2}\,,\qquad \epsilon^{(0)}(q)=(1,0,0,0)\,, \qquad (2.2)$$

and

$$\epsilon^{(a)}(-q)=(-)^a\epsilon^{(a)}(q).$$

It is convenient to define crossing (in ν) even amplitudes

$$S^{ab}(\nu,q^2)=T^{ab}(\nu,q^2)+T^{-b-a}(\nu,q^2)$$

and odd amplitudes

$$A^{ab}(\nu,q^2)=T^{ab}(\nu,q^2)-T^{-b-a}(\nu,q^2)\,.$$

Thus there are six possible classes of amplitudes:

$$S^{++},\ S^{+-},\ S^{00},\ S^{+0};\ A^{++},\ A^{+0}.$$

We also note that under the interchange of μ,ν indices in (2.1), S^{++}, S^{+-}, S^{00}, and A^{+0} are even, and S^{+0} and A^{++} are odd.

To derive the scaling behavior of these amplitudes we follow Bjorken's original procedure[1] of applying the BJL (Bjorken-Johnson-Low) limit to various dispersive representations of T^{ab}. It may be shown (see Appendix B) that such helicity amplitudes (forward scattering involving pairwise identical particles) are free of kinematic singularities in ν (although not in q^2). We learn whether a subtracted or an unsubtracted dispersion relation holds by exploiting the O(4) symmetry of forward scattering, which tells us that in the Regge limit of $\nu\to\infty$ with q^2 fixed,

$$T^{ab}\sim\nu^{\alpha(0)-|M-|a-b||}\,. \qquad (2.3)$$

M is the so-called Toller quantum number of the trajectory α. (This behavior is most readily seen in the Feynman-van Hove model.[18]) The leading natural-parity trajectory is the Pomeranchukon with $\alpha_P(0)=1$; the present experimental data[19] on the (real) photon inclusive reaction $\gamma p\to\pi^- X$ appear to indicate that it is largely $M=0$.[20] We shall also set $M=0$ for the leading unnatural-parity trajectory $\tilde\alpha$ with $\tilde\alpha(0)<1$. Thus S^{++}, S^{+-}, S^{00}, and A^{+0} receive contributions only from natural-parity trajectories, while the Regge asymptotic behavior of S^{+0} and A^{++} is determined by $\tilde\alpha$. Cuts will not change our conclusion.[21]

The dispersion representations for S^{++} and S^{00} thus require one subtraction, while those for S^{+0} and S^{+-} may be unsubtracted. Even though $A^{++}\sim\nu^{\tilde\alpha}$ and $A^{+0}\sim\nu^{\alpha_P-1}$, as odd amplitudes they also satisfy unsubtracted dispersion relations.

$$S^{++}(q^2,\nu)=S^{++}(q^2,\nu=0)+\frac{\nu^2}{\pi}\int_{\nu_0^2}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}S^{++}(q^2,\nu')}{\nu'^2-\nu^2} \qquad (2.4a)$$

$$= S^{++}(q^2, \omega=\infty) + \frac{1}{\pi}\int_0^1 d\omega'^2 \frac{\text{Im}S^{++}(\omega', q^2)}{\omega^2-\omega'^2}, \quad (2.4\text{b})$$

$$S^{+0}(q^2, \nu) = \frac{1}{\pi}\int_{\nu_0{}^2}^\infty d\nu'^2 \frac{\text{Im}S^{+0}(q^2, \nu')}{\nu'^2-\nu^2} \quad (2.5\text{a})$$

$$= \frac{\omega^2}{\pi}\int_0^1 \frac{d\omega'^2}{\omega'^2}\frac{\text{Im}S^{+0}(q^2, \omega')}{\omega^2-\omega'^2}, \quad (2.5\text{b})$$

$$A^{++}(q^2, \nu) = \frac{\nu}{\pi}\int_{\nu_0{}^2}^\infty \frac{d\nu'^2}{\nu'}\frac{\text{Im}A^{++}(q^2, \nu')}{\nu'^2-\nu^2} \quad (2.6\text{a})$$

$$= \frac{\omega}{\pi}\int_0^1 \frac{d\omega'^2}{\omega'}\frac{\text{Im}A^{++}(q^2, \omega')}{\omega^2-\omega'^2}. \quad (2.6\text{b})$$

As usual $\omega = -q^2/2\nu$ and $\omega' = -q^2/2\nu'$. We also wrote $\text{Im}\,T^{ab}(q^2, \nu') = \text{Im}\,T^{ab}(q^2, \omega')$. The dispersion representations for S^{00}, S^{+-}, and A^{+0} have the same form as S^{++}, S^{+0}, and A^{++}, respectively.

We take the BJL limit: $q_0 \to i\infty$ with $\vec{q} = 0$. Since $\epsilon_\mu^{(a)}$ has no time component, only $T_{ij}(q^0, \vec{q}=0)$ enters in $\lim_{q_0\to i\infty} T^{ab}$. We assume that

$$T_{ij} \to \frac{a_{ij}}{q_0} + \frac{b_{ij}}{q_0{}^2} \text{ in the BJL limit,}$$

where a_{ij} and b_{ij} are, respectively, antisymmetric and symmetric in i and j. Hence in the BJL limit

$$S^{++},\ S^{00},\ S^{+-},\ \text{and } A^{+0} \to O(q_0{}^{-2}) \quad (2.7\text{a})$$

and

$$S^{+0} \text{ and } A^{++} \to O(q_0{}^{-1}). \quad (2.7\text{b})$$

Combining Eqs. (2.4)–(2.7) we obtain in the scaling limit, $-q^2 \to \infty$ (recall that $\omega \approx -\frac{1}{2}q_0$ in this limit),

$$\text{Im}\,T^{ab}(q^2, \omega) \sim |q^2|^{-|a-b|/2} F^{ab}(\omega), \quad (2.8)$$

as promised.

Comment: For $a \neq b$, $\text{Im}\,T^{ab}$ is not positive semidefinite. For example, it may happen that $\text{Im}S^{+0}$ scales but

$$\int_0^1 dw\, w^{-2}\,\text{Im}S^{+0}(w) = 0.$$

In Sec. III we show that this is an unlikely possibility from the point of view of the light-cone expansion.

Comment: For the semi-inclusive electroproduction process (A), the above procedure is apparently not applicable since we would have to write dispersion relations in the missing-mass variable. This is in fact the only obstacle for an identical derivation of Eq. (1.2) for process (A).

III. LIGHT-CONE DOMINANCE IN A GENERAL ELECTROPRODUCTION PROCESS

It is well-known that for electroproduction process (B) where no hadron is detected W^{ab} as shown in Eq. (1.1) is directly related to the cross section, and its behavior in the deep-inelastic region in momentum space reflects the light-cone structure[22] of the product of two currents in configuration space. In this section we shall concentrate on the possible scaling behavior for structure functions of the semi-inclusive process (A) in the Bjorken limit. It has been argued[23,24] recently that such a behavior may also be deduced from light-cone dominance of a forward amplitude as the one in Eq. (1.1). We shall not present in this paper a detailed discussion of the kinematics of a general coincidence electroproduction process, but refer the reader to Ref. 3. We note that the structure function for process (A) is only a piece of the total absorptive forward Compton amplitude (1.1): It is the semiconnected part corresponding to the discontinuity in the missing-mass variable (with all other variables fixed.) It has been conjectured[4] that scaling behavior of this semiconnected part is the same as the full absorptive amplitude, and that therefore its scaling behavior may be deduced by examining directly the light-cone structure of (1.1). Callan and Gross[24] have shown that if the leading bilocal operators appearing in the light-cone expansion are in fact products of local operators (this is the case for free-quark theory[25]), then this conjecture is indeed true – they have shown that such a light-cone structure contains the desired semiconnected part. It should be noted that this approach to the question of scaling in semi-inclusive process is in practice equivalent to the multilocal light-cone expansion put forward by Ellis[23]: In effect both approaches allow one, in the Bjorken limit, to move particles from out-state into in-state and hence relate by closure the structure functions to an expression as Eq. (1.1). Accepting this argument the procedure to derive scaling is then identical to the one we would employ in deriving scaling for the simpler process (B).

We parametrize the Bjorken limit by taking

$$q^\mu = (\xi, 0, 0, \xi + m\omega_1) \quad (3.1)$$

with $\xi \to \infty$ and all other momenta $\{p_i\}$ fixed with p_1 at rest. We have in this limit

$$q^2 = O(\xi), \quad qp_i = O(\xi), \quad \epsilon^{(\pm)}p_i = O(\xi^0),$$

and

$$\epsilon^{(0)}p_i = O(\xi^{1/2}). \quad (3.2)$$

As we shall see, our result (1.2) is related to the fact that polarization vectors have different asymptotic limits as given in (3.2).

To derive Eq. (1.2) we begin with the general light-cone expansion in terms of bilocal operators:

$$J_\mu(x)J_\nu(0) \underset{x^2\approx 0}{=} g_{\mu\nu} S_1(x,0)(x^2-ix_0\epsilon)^{-2} + x_\mu x_\nu S_2(x,0)(x^2-ix_0\epsilon)^{-3} + (x_\mu g_{\nu\lambda}+x_\nu g_{\mu\lambda}) V_+^\lambda(x,0)(x^2-ix_0\epsilon)^{-2}$$
$$+(x_\mu g_{\nu\lambda}-x_\nu g_{\mu\lambda}) V_-^\lambda(x,0)(x^2-ix_0\epsilon)^{-2} + \epsilon_{\mu\nu\sigma\lambda} x^\sigma A^\lambda(x,0)(x^2-ix_0\epsilon)^{-2} + T_{\mu\nu}(x,0)(x^2-ix_0\epsilon)^{-1} + \cdots, \tag{3.3}$$

where ••• indicates less singular terms. The bilocal operators can of course be expanded in terms of a tower of local operators. We have assumed scale invariance at small distances[26] and that the leading operators have twist equal to two [twist $(\tau) \equiv$ dimension (d) − spin (J)].[27] We have also restricted ourselves only to local operators belonging to the (j,j) representation of the Lorentz group (thus they correspond to the $M=0$ case of Sec. II). Noncanonical dimensions[26] may be readily accommodated by multiplying the right-hand side of (3.3) by a factor $(x^2-ix_0\epsilon)^{-\delta}$. Equation (1.2) would then read $W^{ab}\sim |q^2|^{\delta-|a-b|/2}$ in the scaling region.

As an illustration consider the contribution of the S_2 term. We have the matrix element of the bilocal operator. [For notational simplicity we shall only consider the process (A) with an unpolarized target, and no final polarization is measured. It will be obvious that this restriction may be trivially relaxed.]

$$\langle \{p_i\}\, in |\, S_2(x,0)\,|\{p_i\}\, in\rangle \equiv \tilde{\mathcal{S}}_2(\{xp_i\},\{p_ip_j\})$$
$$\equiv \int \prod_{l=1}^{n} d\alpha_l \exp\left(i\sum_{m=1}^{n}\alpha_m p_m x\right) S_2(\{\alpha_i\},\{p_ip_j\}). \tag{3.4}$$

Substituting into (1.1) we have $(\gamma^\mu = q^\mu + \sum_{m=1}^{n}\alpha_m p_m^\mu)$

$$W^{ab} = \epsilon_\mu^{(a)}\epsilon_\nu^{(b)*}\int \prod_l^n d\alpha_l \mathcal{S}_2 \int d^4x\, e^{i\gamma x} x^\mu x^\nu (x^2-ix_0\epsilon)^{-3}$$
$$\sim \xi^{-1}\int \prod_l^n d\alpha_l\, \mathcal{S}_2\left(1-\sum_m^n \alpha_m\omega_m^{-1}-i\epsilon\right)^{-1}\sum_i \alpha_i(\epsilon^{(a)}p_i)\sum_j \alpha_j(\epsilon^{(b)}p_j)$$
$$\sim \xi^{-(|a|+|b|)/2} F^{ab}(\{w_i\},\{p_ip_j\}), \tag{3.5}$$

where in the last line we have used asymptotic properties of polarization vectors as shown in (3.2).

The contributions by all the bilocal operators to the various helicity structure functions are summarized in Table I. Collecting the leading terms, we immediately obtain Eq. (1.2).

Remark: We note that the axial bilocal $A^\lambda(x,0)$ in (3.3) gives a contribution to $S^{+0}\sim \xi^{-1}\epsilon^{\mu\nu\lambda\rho}\epsilon_\mu^{(+)}\epsilon_\nu^{(0)}q_\lambda \mathfrak{A}_\rho$, where $\mathfrak{A}_\rho$ is some axial vector formed from the available momenta (and polarization vector). Naive power counting would mislead us to the conclusion $S^{+0}\sim O(\xi^{1/2})$. However, explicitly we have

$$\epsilon^{\mu\nu\lambda\rho}\epsilon_\mu^{(+)}\epsilon_\nu^{(0)}q_\lambda\mathfrak{A}_\rho = (\epsilon_3^{(0)}q_0-\epsilon_0^{(0)}q_3)(\epsilon_1^{(+)}\mathfrak{A}_2+\epsilon_2^{(+)}\mathfrak{A}_1)$$
$$=(-q^2)^{1/2}(\epsilon_1^{(+)}\mathfrak{A}_2+\epsilon_2^{(+)}\mathfrak{A}_1), \tag{3.6}$$

and thus $S^{+0}\sim O(\xi^{-1/2})$ as shown in Table I. We shall have further occasion to see manifestation of such a purely kinematical cancellation when discussing deep-inelastic scattering on a spin-1 target in the next section [see Eq. (4.8)].

TABLE I. The contributions by the bilocal operators to the various helicity structure functions.

	S^{++}	S^{+-}	A^{00}	A^{+0}	A^{++}	S^{+0}
S_1	ξ^0		ξ^0			
S_2	ξ^{-1}	ξ^{-1}	ξ^0	$\xi^{-1/2}$		
V_+	ξ^{-1}	ξ^{-1}	ξ^0	$\xi^{-1/2}$		
T	ξ^{-1}	ξ^{-1}	ξ^0	$\xi^{-1/2}$		
V_-					ξ^{-1}	$\xi^{-1/2}$
A					ξ^0	$\xi^{-1/2}$

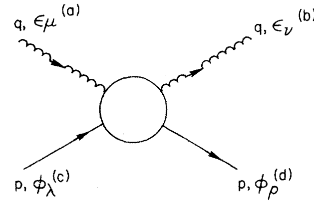

FIG. 4. Kinematical notation for deep-inelastic scattering on spin-1 target.

With the notation we have established, it is easy to see that if the current commutator on the light cone is the one given in the canonical quark-gluon model [see Eq. (4.3) below] the generalized Callan-Gross relation $W^{00}/W^{++} \to 0$ as $\xi \to \infty$ holds for the general process (A) and (B). This is hardly surprising from the point of view of the parton model[28] since the Callan-Gross relation follows from the property of the virtual-photon–parton–parton vertex.

IV. QUARK-MODEL LIGHT-CONE ALGEBRA AND DEEP-INELASTIC SCATTERING ON A SPIN-ONE TARGET

In this section we illustrate our previous discussion by considering in some detail deep-inelastic scattering on a (polarized) spin-1 target.

The kinematics of a forward Compton scattering on spin-1 target is illustrated in Fig. 4. The absorptive part is ($\nu = q \cdot p$)

$$\begin{aligned} W_{cd}^{ab}(q^2, \nu) &= \int d^4x\, e^{iqx} \langle p, \phi^{(d)} | [J^\mu(x), J^\nu(0)] | p, \phi^{(c)} \rangle \epsilon_\mu^{(a)} \epsilon_\nu^{(b)*} \\ &= W^{\mu\nu\lambda\rho}(q,p) \epsilon_\mu^{(a)} \epsilon_\nu^{(b)*} \phi_\lambda^{(c)} \phi_\rho^{(d)*} \end{aligned} \tag{4.1}$$

with

$$\begin{aligned} W^{\mu\nu\lambda\rho} &= (-W_1 g^{\mu\nu} + W_2 p^\mu p^\nu) g^{\lambda\rho} + (-W_3 g^{\mu\nu} + W_4 p^\mu p^\nu) q^\lambda q^\rho + W_5 (g^{\mu\lambda} g^{\nu\rho} + g^{\mu\rho} g^{\nu\lambda}) \\ &\quad + W_6 (p^\mu g^{\lambda\nu} q^\rho + p^\nu g^{\lambda\mu} q^\rho + p^\mu g^{\nu\rho} q^\lambda + p^\nu g^{\mu\rho} q^\lambda) + W_7 (g^{\mu\lambda} g^{\nu\rho} - g^{\mu\rho} g^{\nu\lambda}) \\ &\quad + W_8 (p^\mu g^{\lambda\nu} q^\rho - p^\nu g^{\lambda\mu} q^\rho - p^\mu g^{\nu\rho} q^\lambda + p^\nu g^{\mu\rho} q^\lambda), \end{aligned} \tag{4.2}$$

where terms proportional to q^μ, q^ν, p^λ, or p^ρ have been dropped. The invariant amplitudes $\{W_i\}$ are, of course, functions of q^2 and ν; their relations to the helicity amplitudes $\{W_{cd}^{ab}\}$ are explicitly given in Appendix C.

We assume that the light-cone commutator between currents has the structure given by the free-quark model,

$$[J_\mu(x), J_\nu(0)] \underset{x^2 \approx 0}{=} [S_{\mu\nu\sigma\tau} V^\sigma(x,0) + \epsilon_{\mu\nu\sigma\tau} A^\sigma(x,0)] \partial^\tau (\epsilon(x^0)\delta(x^2)) + \cdots, \tag{4.3}$$

where $S_{\mu\nu\sigma\tau} = g_{\mu\sigma} g_{\nu\tau} + g_{\mu\tau} g_{\nu\sigma} - g_{\mu\nu} g_{\sigma\tau}$.

The calculation leading to scaling behavior of $\{W_i\}$ is straightforward. We introduce the bilocal matrix elements

$$\begin{aligned} \langle p, \phi^{(d)} | V_\sigma(x,0) | p, \phi^{(c)} \rangle &= p_\sigma [(\phi^{(c)} \cdot \phi^{(d)*}) \tilde{f}_1 + (\phi^{(c)} \cdot x)(\phi^{(d)*} \cdot x) \tilde{f}_2] \\ &\quad + x_\sigma [(\phi^{(c)} \cdot \phi^{(d)*}) \tilde{g}_1 + (\phi^{(c)} \cdot x)(\phi^{(d)*} \cdot x) \tilde{g}_2] + \tfrac{1}{2} [(\phi^{(c)} \cdot x) \phi_\sigma^{(d)*} + (\phi^{(d)*} \cdot x) \phi_\sigma^{(c)}] \tilde{h} \end{aligned} \tag{4.4a}$$

and

$$\langle p, \phi^{(d)} | A^\sigma(x,0) | p, \phi^{(c)} \rangle = i\epsilon^{\sigma\alpha\beta\gamma} \phi_\alpha^{(c)} \phi_\beta^{(d)*} (p_\gamma \tilde{f}_A + x_\gamma \tilde{g}_A), \tag{4.4b}$$

where $\tilde{f}_{1,2,A}$, $\tilde{g}_{1,2,A}$, and $\tilde{h}$ are functions of $x \cdot p$ (and p^2). Taking Fourier transform of these functions and inserting them into Eqs. (4.1)–(4.4) one readily obtains in the scaling region

$$\begin{aligned} &W_1 \sim -\tfrac{1}{2} f_1 \equiv F_1, \quad \nu W_5 \sim -\tfrac{1}{2} h \equiv F_5, \\ &\nu W_2 \sim -\omega f_1 \equiv F_2, \quad \nu^2 W_6 \sim -\tfrac{1}{4}(2f_2' - \omega h' - h) \equiv F_6, \\ &\nu^2 W_3 \sim \tfrac{1}{2}(f_2'' + h') \equiv F_3, \quad W_7 \sim \tfrac{1}{2} f_A \equiv F_7, \\ &\nu^3 W_4 \sim \omega f_2'' + 2f_2' \equiv F_4, \quad \nu W_8 \sim \tfrac{1}{2} f_A \equiv F_8, \end{aligned} \tag{4.5}$$

where $F_i = F_i(\omega, p^2)$. By inspection, we obtain four relations among the scaling functions:

$$F_1 = \frac{1}{2\omega} F_2, \tag{4.6}$$

$$F_3 = \frac{1}{2\omega}(F_4 + 2F_5 + 4F_6), \tag{4.7}$$

$$F_7 = F_8, \tag{4.8}$$

and also

$$\omega \frac{\partial^2 F_5}{\partial \omega^2} + 4 \frac{\partial F_5}{\partial \omega} + 2 \frac{\partial F_6}{\partial \omega} + 2F_3 = 0. \tag{4.9}$$

Equation (4.6) is just the Callan-Gross relation. Equation (4.7) looks less familiar but is in fact also a "Callan-Gross" relation. It is easy to see why. Consider the general case of scattering on a spin-J target. The argument of Callan-Gross leads to the result that the spin-averaged longitudinal structure function vanishes in the scaling limit, i.e., $\sum_{m=-J}^{J} W_{mm}^{00} \to 0$. By positivity we in fact have $W_{mm}^{00} \to 0$ for each m. After using parity invariance we can conclude that for integer-spin target there are in general $(J+1)$ independent Callan-Gross relations, and for half-integer-spin target, $(J+\frac{1}{2})$ relations. The relation in Eq. (4.8) differs from the rest in that it is "purely kinematical." We have such a result whenever the axial-vector bilocal operator is present in the light-cone expansion. This has already been explained in Sec. III [see Eq. (3.6)]. The relation in Eq. (4.9) will be discussed in the next section. When the scaling behavior found in Eq. (4.5), together with Eqs. (4.6)–(4.8), is translated into the language of helicity structure functions (see Appendix C), it corresponds exactly to our general result Eq. (1.2) with the added quark-model result of $F^{00}(\omega)=0$. The lesson is that helicity structure functions have many advantages over the invariant structure functions: The cross sections, the angular and spin correlation, the positivity conditions, the Callan-Gross relation, and, most of all, our result Eq. (1.2), when expressed in terms of helicity structure functions, take on simple forms for the whole class of inclusive and semi-inclusive electroproduction processes.

Our intention is to apply the discussion of this section to the two-photon process $e^\pm e^- \to e^\pm e^- + X$. Many authors[13] have proposed that this process be used to measure deep-inelastic scattering on a virtual photon target and have given cross-section estimates. That the light cone may be relevant has been argued in the literature[23] (see Sec. III). For simplicity we assume that operator Schwinger terms are absent so that the amplitude for the two-photon process is proportional to $\langle X|T(J_\mu(x)J_\nu(0))|0\rangle$. Referring to the cross section given for example in Eq. (28) of Ref. 29 we see that the scaling behavior in Eq. (1.2) leads to the vanishing of terms proportional to $\cos\chi$ and $\cos 2\chi$ in the scaling region. (χ is the azimuthal angle defined in Ref. 29.)

V. LIGHT-CONE EXPANSION BEYOND TWIST TWO

In the light-cone analysis presented in Sec. III we allow only local operators that belong to the (j,j) representation of O(3, 1), i.e., the bilocal operators appearing in the expansion have only symmetric indices. With this restriction the twist classification is clearly useful since the tower with the lowest twist dominates in Eq. (3.3).

In general, however, local fields belonging to (j,j') with $j\neq j'$ may well contribute. We refer to these fields as having $M\equiv|j-j'|\neq 0$. In practice this means we will have to allow bilocal operators with antisymmetric indices (the notation $[\alpha,\beta,\gamma,\ldots]$ indicates that the tensor is totally antisymmetric in these indices):

$$\begin{aligned} J^\mu(x)J^\nu(0) \underset{x^2\approx 0}{=} & [M=0 \text{ terms as in Eq. (3.3)}] + U_1^{[\mu,\nu]}(x,0)(x^2-ix_0\epsilon)^{-1+\delta_1} \\ & + x_\lambda U_2^{[\mu,\lambda]\nu}(x,0)(x^2-ix_0\epsilon)^{-1+\delta_2} \pm (\mu \longleftrightarrow \nu) + x^\mu x_\lambda U_3^{[\nu,\lambda]}(x,0)(x^2-ix_0\epsilon)^{-2+\delta_3} \pm (\mu \longleftrightarrow \nu) \\ & + x_\sigma x^\rho \epsilon^{\mu\nu}{}_{\lambda\rho} U_4^{[\lambda,\delta]}(x,0)(x^2-ix_0\epsilon)^{-2+\delta_4} + x_\lambda x_\sigma V^{[\mu,\lambda][\nu,\sigma]}(x,0)(x^2-ix_0\epsilon)^{-1+\kappa}. \end{aligned} \tag{5.1}$$

The parameters δ_i and κ are related to the twist of the local operators appearing in U_i and V by $\tau(U_i)=3+2\delta_i$ and $\tau(V)=4+2\kappa$. Just as in Sec. III one readily works out the contribution of these $M\neq 0$ operators to the structure functions in the scaling region (Table II).

We observe that for $\delta_i=\kappa=0$ the twist-three operators U_i and twist-four operator V can already compete with the leading $M=0$ twist-two operators given in Sec. II in their contributions to the kinematically suppressed helicity-flip structure functions W^{+0} and W^{+-}. Hence the presence of such terms will in general invalidate any relation involving scaling functions F^{ab} with $a\neq b$ when it is derived using only a twist-two light-cone expansion. This is the case with our relation Eq. (4.9) and the relation (4.12) of Ref. 3. Thus the condition for their validity is precisely the absence of $M\neq 0$ fields of twist three and four of the appropriate form as shown above (see also Appendix A). It is of interest to see if the experimental data support this assumption. (We note that such terms are absent in the light-cone expansion in a free-quark model.) We prefer to say that this phenomenon of $M\neq 0$ operators competing with $M=0$ opera-

TABLE II. Contribution of the $M\neq 0$ operators U_i and V to the structure functions in the scaling region. ξ is the scaling parameter introduced in Sec. III.

	W^{++}	W^{00}	W^{+0}	W^{+-}
U_i	$\xi^{-1-\delta_i}$	$\xi^{-1-\delta_i}$	$\xi^{-1/2-\delta_i}$	$\xi^{-1-\delta_i}$
V	$\xi^{-1-\kappa}$	$\xi^{-2-\kappa}$	$\xi^{-3/2-\kappa}$	$\xi^{-1-\kappa}$

tors of lower twist shows that the twist classification is not a convenient one for $M \neq 0$ field. (The twist of a $M \neq 0$ operator is "abnormally" large because their spin J is less than the number of Lorentz indices they carry.)

Some typical $M \neq 0$ bilocal operators that one may construct formally out of a quark field (with dimension $\frac{3}{2}$) are

$$U_1^{[\alpha,\beta]}(x,0) = \bar{\psi}(x)\sigma^{\alpha\beta}\psi(0)$$

and

$$V^{[\alpha,\beta][\lambda,\delta]}(x,0) = \bar{\psi}(x)(\gamma^\alpha\partial^\beta - \gamma^\beta\partial^\alpha) \times(\gamma^\lambda\partial^\delta - \gamma^\delta\partial^\lambda)\psi(0)\,.$$

They correspond to $\kappa = \delta_1 = -\frac{1}{2}$. (These terms also exemplify the most singular $M \neq 0$ terms which can be built out of fields with naive dimensions.) According to Table II this implies a "kinematical" enhancement: namely, that $W^{+0} \to (\xi^0)$ and $W^{+-} \to O(\xi^{-1/2})$. Now such a factor of $1/\sqrt{x^2}$ cannot pop up in perturbation theory and in the parton models proposed so far but cannot be excluded on general grounds. In perturbation theory the factor of $1/\sqrt{x^2}$ is replaced by m, with m some characteristic mass in the theory (such as the quark mass in some formal quark-gluon model). It is of interest to settle this question by measuring W^{+0} and W^{+-}.

We note that the Callan-Gross relation, if satisfied by the contribution of the $M=0$ operators, is not destroyed by the $M \neq 0$ operators. Also, the SLAC-MIT single-arm experiment measures only W^{++} and W^{00} and hence cannot provide any information on the questions discussed in this section.

To summarize, in a general canonical light-cone expansion (i.e., for the most singular terms we only allow those that may be constructed out of free fields and with integer powers of x^2) our result (1.2) follows. This scaling behavior is not affected by the presence of the $M \neq 0$ terms in such an expansion. However, a derivation of relations involving $F^{ab}(\omega)$ with $a \neq b$ must require definite knowledge of these $M \neq 0$ terms up to (and including) twist four.

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APPENDIX A

The problem of deep-inelastic scattering on a polarized nucleon has already been discussed extensively.[9-12] Our general result Eq. (1.2) when translated into invariant amplitudes reads

$$\nu\,\mathrm{Im}G_1(\nu, q^2) \to g_1(\omega)\,, \qquad \nu^2\,\mathrm{Im}G_2(\nu, q^2) \to g_2(\omega) \tag{A1}$$

in the scaling region. Here G_1 and G_2 are the usual spin-dependent structure functions.[30] This scaling behavior has been presented in the literature.[10-12]

We note that both G_1 and G_2 satisfy unsubtracted dispersion relations and in the BJL limit they vanish as q_0^{-2} and q_0^{-5}, respectively.[31] Thus the original Bjorken method of deriving scaling (see Sec. II) will also lead to (A1), contrary to the conclusion reached in Ref. 10.

Hey and Mandula[11] have presented the following sum rule as a prediction of the quark-model light-cone expansion:

$$\int_0^1 d\omega\, g_2(\omega) = 0\,. \tag{A2}$$

We would like to point out that this, being a relation involving the kinematically suppressed W^{+0},[32] is a highly model-dependent result. In particular, it is sensitive to contributions from $M \neq 0$ terms of higher twist, for example, the U_1 term in Eq. (5.1) with $\delta_1 = 0$. Thus this sum rule is on the same footing as Eq. (4.9) of Sec. IV [and Eq. (4.12) of Ref. 3]. Associated with this problem is the question of maintaining manifest gauge invariance. The quark-model expansion as presented in Eq. (4.3) is not manifestly gauge-invariant. Usually this does not introduce any ambiguity since for gauge completion we need only add operators that may differ from each other by higher twist terms.[33] However, it does make a difference for the validity of (A2) since G_2 is kinematically suppressed. For example, adding the twist-three term

$$\epsilon(x^0)\delta(x^2)\epsilon_{\mu\nu\lambda\sigma}\partial^\lambda A^\sigma(x,0)$$

to Eq. (4.3) will lead to (A2). On the other hand, one may complete the gauge by applying the usual projector operator

$$\left(g_{\mu\mu'} - \frac{q_\mu q_{\mu'}}{q^2}\right)\left(g_{\nu\nu'} - \frac{q_\nu q_{\nu'}}{q^2}\right)$$

to the result obtained from Eq. (4.3). This procedure[33] formally corresponds to adding various extra terms to Eq. (4.3) and will spoil (A2). We know of no general argument for preferring either method to implement gauge invariance. The question boils down to what extent one may reliably extract information from a model.

APPENDIX B

Here we shall show that the forward Compton helicity amplitudes T_{cd}^{ab} as used in Sec. II are free of kinematical singularities in $\nu = p\cdot q$. [This is manifestly true for the cases of spin-$\frac{1}{2}$ and spin-1 targets as illustrated in Ref. 32 and Eq. (C2).]

$$T^{ab}_{cd}(\nu, q^2) = \epsilon^{(a)}_{\mu}(q)\epsilon^{(b)*}_{\nu}(q)\phi^{(c)}_{\{\alpha\}}(p) \times \phi^{(d)*}_{\{\beta\}}(p)T^{\mu\nu\{\alpha\}\{\beta\}}_{(q,p)}\ , \quad \text{(B1)}$$

where $\{\alpha\} = \alpha_1 \cdots \alpha_J$ and $\phi^{(c)}_{\{\alpha\}}(p)$ denotes the polarization tensor of the spin-J target in helicity state (c). (For the time being we shall restrict ourselves to cases where J is an integer.) In terms of invariant amplitudes we have

$$T^{\mu\nu\{\alpha\}\{\beta\}}_{(q,p)} = \sum_i L_i{}^{\mu\nu\{\alpha\}\{\beta\}}_{(q,p)}\ T_i(\nu, q^2)\ , \quad \text{(B2)}$$

$\{L_i{}^{\mu\nu\{\alpha\}\{\beta\}}_{(q,p)}\}$ being a set of invariant bases constructed out of $g_{\lambda\rho}$, $\epsilon_{\lambda\rho\sigma\delta}$ and a minimal number of q's and p's; the invariant amplitudes $\{T_i\}$ are expected to be free of kinematical singularities. Thus our task is to show that the products

$$M^{ab}_{cd}(i,p,q) \equiv \epsilon^{(a)}\epsilon^{(b)*}\phi^{(c)}\phi^{(d)*}L_i$$

do not have any singularities in ν. For this purpose it is convenient to write explicitly in the rest frame of the target

$$p_\mu = (m, 0, 0, 0)\ ,$$
$$q_\mu = [\nu, 0, 0, (\nu^2 - q^2)^{1/2}]\ , \quad \text{(B3)}$$
$$\epsilon^{(0)}_\mu = [(\nu^2 - q^2)^{1/2}, 0, 0, \nu]/(-q^2)^{1/2}\ .$$
$$\epsilon^{(\pm)}_\mu = -(0, \pm 1, i, 0)/\sqrt{2}\ .$$

Hence

$$\phi_{\alpha_1\cdots\alpha_J} = 0 \ \text{ for any } \alpha_i = 0\ , \quad \text{(B4)}$$

and also

$$p\cdot\epsilon^{(\pm)} = 0\ . \quad \text{(B5)}$$

Clearly there cannot be any ν-dependent factors in the denominator of M^{ab}_{cd}. The only nontrivial step left is to show that they are also free of square-root singularities; namely, in $M^{ab}_{cd}(i,p,q)$ the total number of $\epsilon^{(0)}_0$ and q_3 of (B3) is always even. This may be demonstrated by direct inspection [and using Eqs. (B4) and (B5)] of the four distinctive types of terms in M:

$$\phi^{(c)}_{\{\alpha_1\cdots\alpha_J\}}\phi^{(d*)}_{\{\alpha_{J+1}\cdots\alpha_{2J}\}}[f_1\epsilon^{(a)}\cdot\epsilon^{(b)*}q^{\alpha_1}\cdots q^{\alpha_{2J}} + f_2\epsilon^{(a)}\cdot p\epsilon^{(b)*}\cdot pq^{\alpha_1}\cdots q^{\alpha_{2J}} + f_3\epsilon^{(a)}\cdot p\epsilon^{(b)*\alpha_1}q^{\alpha_2}\cdots q^{\alpha_{2J}} + f_4\epsilon^{(a)\alpha_1}\epsilon^{(b)*\alpha_2}q^{\alpha_3}\cdots q^{\alpha_{2J}}]\ . \quad \text{(B6)}$$

Other terms differ from the above four by permutation of indices $\{\alpha\}$ or differ by an even number of q_3's when $\{\alpha\}$ are contracted.

The above argument may be extended in a straightforward manner to cases of $J' = J+\frac{1}{2}$ = half integers if a Rarita-Schwinger[34] type of formalism is used. $\phi_{\{\alpha\}}$ is then the polarization tensor-spinor: $(\not p - m)\phi_{\{\alpha\}} = 0$. Besides (B6) we must also consider terms with $\phi^{(d)*}_{\{\beta\}}\phi^{(c)}_{\{\alpha\}}$ replaced by $\bar\phi^{(d)}_{\{\beta\}}\gamma_\lambda\gamma_5\phi^{(c)}_{\{\alpha\}}$ and also the totally antisymmetric tensor $\epsilon_{\mu\nu\lambda\sigma}$ must appear. We shall leave to the reader as an exercise to show that again $\epsilon^{(0)}_0$ and q_3 appear in even numbers in these cases. In fact it is obvious that our conclusion continues to hold when the photon is replaced by any other particle.[35]

APPENDIX C

The helicity structure functions $W^{ab}_{cd}(q^2, p^2, \nu = p\cdot q)$ of a spin-1 particle are related to invariant amplitudes of Eq. (4.1) by

$$W^{ab}_{cd} = \epsilon^{(a)}_\mu(q)\epsilon^{(b)*}_\nu(q)\phi^{(c)}_\lambda(p)\phi^{(d)*}_\rho(p)\,W^{\mu\nu\lambda\rho}(q,p)\ . \quad \text{(C1)}$$

Explicitly we have

$$S^{++}_{+} \equiv \tfrac{1}{2}(W^{++}_{++} + W^{--}_{++}) = -W_1 + W_5\ ,$$

$$S^{++}_{0} \equiv W^{++}_{00} = W_1 + \left(\frac{q^2p^2 - \nu^2}{p^2}\right)W_3\ ,$$

$$S^{00}_{+} \equiv W^{00}_{++} = W_1 - \left(\frac{q^2p^2 - \nu^2}{q^2}\right)W_2\ ,$$

$$S^{00}_{0} \equiv W^{00}_{00} = -W_1 + \left(1 - \frac{\nu^2}{p^2q^2}\right)[p^2W_2 - q^2W_3 + (p^2q^2 - \nu^2)W_4] + \frac{2\nu^2}{p^2q^2}\left(W_5 - \frac{2}{\nu}(p^2q^2 - \nu^2)W_6\right)\ , \quad \text{(C2)}$$

$$S^{+-} \equiv W^{+-}_{+-} = 2W_5\ ,$$

$$S^{+0} \equiv \tfrac{1}{2}(W^{+0}_{+0} + W^{0-}_{+0}) = \frac{1}{(p^2q^2)^{1/2}}[\nu W_7 + (p^2q^2 - \nu^2)W_8]\ ,$$

$$A^{++} \equiv \tfrac{1}{2}(W^{++}_{++} - W^{--}_{++}) = W_7\ ,$$

$$A^{+0} \equiv \tfrac{1}{2}(W^{+0}_{+0} - W^{0-}_{+0}) = \frac{1}{(p^2q^2)^{1/2}}[\nu W_5 - (p^2q^2 - \nu^2)W_6]\ .$$

We have used the notation of S^{ab} and A^{ab} of Sec. II and, of course, here they are absorptive parts of the Compton amplitudes. In the deep-inelastic limit, our result Eq. (4.5) gives the following scaling behavior for the helicity structure functions ($\omega = -q^2/2\nu$):

$$A^{++} \to F_7 ,$$

$$S_+^{++} \to -F_1 ,$$

$$S_0^{++} \to F_1 - F_3 ,$$

$$S_+^{00} \to F_1 - \frac{1}{2\omega} F_2 ,$$

$$S_0^{00} \to -\left(F_1 - \frac{1}{2\omega} F_2\right) - \frac{1}{p^2}\left(F_3 - \frac{1}{2\omega}(F_4 + 2F_5 + 4F_6)\right),$$

$$\frac{1}{\sqrt{\nu}} S^{+0} \to \frac{1}{(2\omega p^2)^{1/2}} (F_7 - F_8) , \qquad \text{(C3)}$$

$$\sqrt{\nu}\, A^{+0} \to \frac{1}{(2\omega p^2)^{1/2}} (F_5 + F_6) ,$$

$$\nu S^{+-} \to 2F_5 .$$

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$$\tfrac{1}{2}\left(W^{++}_{1/2,1/2} - W^{++}_{-1/2,1/2}\right) = \frac{2}{m^2}\left(p \cdot q\, \mathrm{Im}G_1 + q^2\, \mathrm{Im}G_2\right),$$

$$W^{+0}_{1/2,-1/2} = -2\left(-\frac{q^2}{m^2}\right)^{1/2}\left(\mathrm{Im}G_1 - \frac{p \cdot q}{m^2}\, \mathrm{Im}G_2\right).$$

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