Comments and Addenda

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Heavy leptons in an O(4) gauge model

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Heavy-lepton contributions to the magnetic moment of the muon are calculated in the framework of a unified theory of weak and electromagnetic interactions based on a strict O(4) gauge group. The model, previously reported, incorporates the Han-Nambu quarks and CP violation, and leads to a kinematic $\Delta I = 1/2$ rule for nonleptonic weak processes. This calculation shows that recent experimental results can definitely rule this model out; implications for general models based on the O(4) group are discussed.

A unified theory of weak and electromagnetic interactions based on a spontaneously broken O(4) gauge symmetry has recently been discussed.[1] It belongs to the class of $O(4)\times\mathcal{G}$ models, originally proposed by Pais,[2] which differ from $SU(2)\times U(1)$ theories in that two sets of charged intermediate vector bosons of comparable masses separately mediate $\Delta S=0$ and $\Delta S=1$ semileptonic processes.[3] Since this model is based upon a simple, compact Lie group, there is only one coupling, determined by the fine-structure constant; this model may therefore be properly called a unified theory.

The model satisfies the usual criteria of acceptability for a theory of weak and electromagnetic interactions: It reduces to Cabibbo theory at low energies, it is anomaly-free and hence renormalizable, and the strangeness-changing neutral-current effects are properly suppressed. In addition, it has the following desirable features: The hadron sector of the model is based on three Han-Nambu triplets of "valence quarks" superimposed on an $SU(3)\times SU(3)$ singlet background, the $\Delta I=\frac{1}{2}$ rule for nonleptonic weak processes follows naturally from this quark assignment, and maximal CP-violating phases are allowed in the Lagrangian with all physical effects remaining superweak. (In particular, electric dipole moments are of at least fifth order in e and are compatible with the present stringent experimental upper bounds.[4])

In the original paper[1] it was shown that several predictions of the model would make an early confrontation with experimental results possible. This paper shows that the model is not compatible with the latest lower bound on the mass of a charged heavy muon and the data on the muon magnetic moment and can therefore be ruled out as a viable theory of electromagnetic and weak interactions.

The O(4) model has two sets of charged vector bosons, denoted by $W_1^{\pm}$ and $W_2^{\pm}$, with masses μ_1 and μ_2, respectively, one massive neutral vector boson, and the photon. Since $W_1^{\pm}$ couple to $\Delta S=0$ and $W_2^{\pm}$ couple to $\Delta S=1$ and since there is *one* coupling constant in the Lagrangian, the masses μ_1 and μ_2 are simply related by the Cabibbo angle, θ_C,

$$\tan\theta_C = \mu_1^2/\mu_2^2\,. \tag{1}$$

The lepton sector of the model contains two heavy neutral muons, y_1 and y_2, with masses M_1 and M_2, one massive positive muon, y^+, with mass M_+, the massless muon neutrino, and the muon with mass m. The lepton masses satisfy

$$M_+ + m = M_1 \tag{2}$$

and

$$(1-\tfrac{2}{9}\cos^2\phi)^{1/2}M_2 = M_1\,, \tag{3}$$

where ϕ is the arbitrary mixing between y_2 and ν in the left-handed lepton states. Of course, there is a similar set of electronlike leptons which is not of interest here.

Weak corrections to the muon magnetic moment have been calculated in detail previously for several gauge models.[5] A number of diagrams contribute to the weak corrections in first order in G_F. Although individual diagrams depend upon the gauge used in the calculation, the final result must be gauge-independent. The particle content of these contributions is most easily described in the U gauge in which the would-be Goldstone bosons are gauge-transformed away and appear as the longitudinal components of massive vector bosons. Although there were ambiguities in the original U-gauge calculations, the resolution of these difficulties is now known.[5] Therefore, this discussion will focus on U-gauge contributions.

In a general gauge model three types of diagrams contribute in the U gauge. The muon legs may be directly connected by a neutral lepton line, a neutral vector boson line, or a Higgs scalar line. Since the massive neutral vector boson in this model couples only to neutral leptons, there are no diagrams involving a neutral current in this order. Higgs scalar exchanges are ignored, since these terms are suppressed by factors of the ratio of m to the arbitrary scalar mass and therefore put no constraint on the relevant physical parameters. Therefore, the only contributions of interest here are those involving neutral lepton exchange—the diagrams involving y_1 and y_2 shown in Fig. 1 and diagrams involving neutrino exchange.[6]

The general structure of the lepton–vector-boson coupling appearing in diagrams such as those in Fig. 1 is given by

$$W_\mu \bar{y}[\tfrac{1}{2}\alpha\gamma^\mu(1-\gamma_5)+\tfrac{1}{2}\beta\gamma^\mu(1+\gamma_5)]\mu^- , \qquad (4)$$

where y is the neutral lepton spinor and W_μ is either W_1 or W_2. The spinor factor in these diagrams then becomes

$$\bar{\mu}(p')[\tfrac{1}{2}\alpha^*\gamma^\mu(1-\gamma_5)+\tfrac{1}{2}\beta^*\gamma^\mu(1+\gamma_5)][M+\not{k}] \times[\tfrac{1}{2}\alpha\gamma^\nu(1-\gamma_5)+\tfrac{1}{2}\beta\gamma^\nu(1+\gamma_5)]\,\mu(p), \qquad (5)$$

where p' (p) is the final (initial) muon four-momentum, k is the momentum of the exchange neutral, and M is the neutral-lepton mass. Terms contributing to the muon moment from the $\not{k}$ term are neglected since they are suppressed by a factor of (m/M) relative to the other terms. Diagrams involving neutrino exchange are therefore also ignored relative to the diagrams in Fig. 1. The terms proportional to M in Eq. (5) reduce to

$$M\,\mathrm{Re}\,\alpha\beta^*\bar{\mu}\gamma^\mu\gamma^\nu\mu + iM\,\mathrm{Im}\,\alpha^*\beta\bar{\mu}\gamma^\mu\gamma^\nu\gamma_5\mu . \qquad (6)$$

Contributions to the muon magnetic moment, $F_2(0)$, proportional to M come from the first term in (6); the second term, if nonzero, would contribute to a muon electric dipole moment.

The relevant terms in the currents appearing in the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -ieW_1^\mu j_\mu - ieW_2^\mu j_\mu \qquad (7)$$

in the O(4) model are

$$j_\mu^{(1)} = \frac{1}{\sqrt{2}}(\bar{y}_1\gamma_\mu\mu)_L + \tfrac{1}{3}\cos\phi(\bar{\nu}\gamma_\mu\mu)_L + \frac{(9-2\cos^2\phi)^{1/2}}{3\sqrt{2}}(\bar{y}_2\gamma_\mu\mu)_L + \frac{1}{\sqrt{2}}(\bar{y}_1\gamma_\mu\mu)_R + \frac{1}{\sqrt{2}}(\bar{y}_2\gamma_\mu\mu)_R , \qquad (8)$$

$$\epsilon j_\mu^{(2)} = \frac{1}{\sqrt{2}}(\bar{y}_1\gamma_\mu\mu)_L - \tfrac{1}{3}\cos\phi(\bar{\nu}\gamma_\mu\mu)_L - \frac{(9-2\cos^2\phi)^{1/2}}{3\sqrt{2}}(\bar{y}_2\gamma_\mu\mu)_L + \frac{1}{\sqrt{2}}(\bar{y}_1\gamma_\mu\mu)_R - \frac{1}{\sqrt{2}}(\bar{y}_2\gamma_\mu\mu)_R , \qquad (9)$$

where L and R designate left- and right-handed terms and ϵ is a phase factor equal to $e^{i\pi/4}$ in this model.

Since for each current the left- and right-handed parts are relatively real there is no contribution to the muon electric dipole moment [$\mathrm{Im}\,\alpha^*\beta=0$ in Eq. (6)]. The leading contribution to $F_2(0)$ from the diagrams in Fig. 1 is given by[7]

$$F_2^W(0) = \frac{-e^2 m}{8\pi^2}[M_1 + \tfrac{1}{3}(9-2\cos^2\phi)^{1/2}M_2] \times(1/\mu_1^2+1/\mu_2^2) = \frac{-e^2 mM_+}{4\pi^2\mu_1^2}(1+\tan\theta_C) , \qquad (10)$$

where Eqs. (1) and (3) have been used to simplify the expression, and M_1 has been approximated by

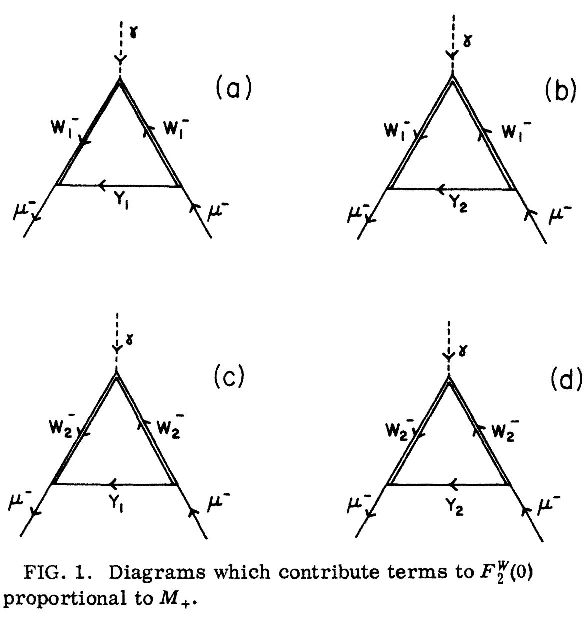

FIG. 1. Diagrams which contribute terms to $F_2^W(0)$ proportional to M_+.

M_+ [see Eq. (2)]. The coupling in Eq. (8) of the neutrino term leads to a relation between the usual weak coupling constant G_F and μ_1^2:

$$\frac{G_F}{\sqrt{2}}\cos\theta_c = \frac{e^2\cos^2\phi}{36\mu_1^2}\,. \tag{11}$$

Since (allowing two standard deviations) the experimental lower bound to the weak corrections to $F_2(0)$ is given by[8]

$$F_2^W(0) \geq -4.7\times 10^{-8}, \tag{12}$$

the mass of the positive muon, M_+, must be substantially less than 1 GeV.

An experimental search for heavy, positively charged muons has recently been reported which sets a lower limit to the mass, M_+, of such a particle as 8 GeV.[9] In this experiment, νp inelastic scatterings were analyzed for events of the type

$$\nu_\mu + p \to y^+ + \text{anything}\,.$$
$$y^+ \searrow \mu^+ + \nu_\mu + \bar{\nu}_\mu\,. \tag{13}$$

The limit on M_+ is obtained by Barish *et al.* by attributing all observed events to such a sequence although the events observed are perfectly consistent with background. It is clear that the neglected terms proportional to m/M_+ could not conspire to bring the upper limit for M_+ obtained here to even order-of-magnitude agreement with the lower limit on M_+ mentioned above. It must therefore be concluded that this O(4) model for weak and electromagnetic interactions is effectively ruled out by this discrepancy.

Any gauge theory based on a strict O(4) gauge symmetry will possess the following properties: Heavy leptons will be present; the currents connecting heavy leptons to muons must be of mixed chirality; even with a modified hadron sector, the vector-boson mass (here $\mu_1 \leq 18$ GeV) will be less than 37 GeV.[2] Thus it seems highly unlikely that the O(4) model can be modified in any reasonable way to avoid having a heavy charged muon with mass less than 8 GeV.[10] Therefore, these results indicate that, despite a number of attractive features, O(4) models do not represent viable theories of weak and electromagnetic interactions.

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[1]T. P. Cheng, Phys. Rev. D 8, 496 (1973).
[2]A. Pais, Phys. Rev. Lett. 29, 1712 (1972); Phys. Rev. D 8, 625 (1973).
[3]G. Segrè, Phys. Rev. 173, 1730 (1968).
[4]A. Pais and J. R. Primack, Phys. Rev. D 8, 3063 (1973); also see discussion below.
[5]See, for example, K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972); J. R. Primack and H. R. Quinn, *ibid.* 6, 3171 (1972).
[6]Because of the "*R* parity" of the O(4) group, the photon does not couple W_1 to W_2. See Ref. 1.
[7]Since the space-time part of the calculation is similar to that in other models, the reader is referred to Ref. 5 for details.
[8]For calculations of the photon contributions to $F_2(0)$ see, for example, E. de Rafael, B. Lautrup, and A. Peterman, Phys. Rep. 3C, 193 (1972); J. Aldins, S. Brodsky, A. Dufner, and T. Kinoshita, Phys. Rev. Lett. 23, 441 (1969); Phys. Rev. D 1, 2378 (1970); M. Levine and J. Wright, Phys. Rev. Lett. 26, 1351 (1971); Phys. Rev. D 8, 3171 (1973). The current experimental number is reported by J. Bailey *et al.*, Phys. Lett. 28B, 287 (1968).
[9]B. C. Barish *et al.*, Phys. Rev. Lett. 32, 1387 (1974).
[10]This assumes that leptons are assigned to low-dimensional representations of the group.