SCALING PROPERTIES OF A GAUGE THEORY WITH HAN–NAMBU QUARKS AND CHARGED VECTOR GLUONS☆

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A renormalizable gauge theory of strong and electromagnetic interactions with integrally charged quarks is constructed, and its scaling behavior is analyzed.

Han and Nambu have, for several years now, advocated models with integrally charged quarks [1,12]. Since it is difficult to distinguish these integrally charged quarks from ordinary hadrons, the fact that they have not been definitely observed yet need not be too troubling. Experiments on e^+e^- annihilation [2] have given these models added interest, because they could account for the unexpectedly large cross-section and the large amount of energy carried off by neutral particles [3].

It is therefore of interest to see how these models can be put inside the framework of renormalizable quantum field theory, especially the non-abelian gauge theories suggested by studies of scaling [4]. Also in such theories the vector gluons are necessarily charged. This provides us with the opportunity to assess the behavior of charged spin 1 partons[‡1] in a renormalizable theory.

For simplicity of presentation we shall at first work out an $SU_3' \times U_1$ gauge theory of strong and electromagnetic interactions. The scaling behavior of such a theory will be analyzed qualitatively and, finally, inclusion of weak interactions in such a theory will also be commented upon.

Consider the Lagrangian,

$$L = -\tfrac{1}{4} C^a_{\mu\nu} C^{a\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - \sum_{i=1}^{2} \bar{\ell}_i (\mathrm{i}\not\partial - m_{\ell_i} - g'\not B)\ell_i$$
$$- \sum_{i=1}^{3} \bar{q}_i (\mathrm{i}\not\partial - m_i + g\frac{\lambda^a}{2} \not C^a + g' b_i \not B) q_i \qquad (1)$$
$$+ \sum_{j=1}^{2} \left| (\partial_\mu + \mathrm{i}g\frac{\lambda^a}{2} C_\mu - \tfrac{1}{3} g' B_\mu)\phi_j \right|^2 - V(\phi_1, \phi_2)$$

C^a_μ $(a = 1...8)$ are gauge fields of SU_3'; B_μ is that of U_1. The $q_{\alpha i}$ $(\alpha, i = 1,2,3)$ are the moment of Han–Nambu quarks, each set of $(q_\alpha)_i$ transforms as a triplet under SU_3' with U_1 quantum numbers $b_1 = 2/3$, $b_2 = b_3 = -1/3$ respectively. The $\ell_{1,2}$ are the lepton fields (electrons and muon) coupled only to the U_1 factor. There are two triplets of complex scalar fields, ϕ_1 and ϕ_2.

The potential $V(\phi_1, \phi_2)$ is supposed to lead to spontaneous symmetry breakdown, with ϕ_i acquiring vacuum expectation values of the form $\langle\phi_1\rangle = (\lambda_1, 0, 0)$ and $\langle\phi_2\rangle = (\lambda_2, \lambda_3, 0)$. This leads to eight massive vector gluons (four of which have unit charges). One linear combination of B_μ and C^8_μ remains to massless, corresponding to the residual local U_1 symmetry, which we identify to be the photon field A_μ. The

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‡1 Cleymans and Komen [5] have discussed charged spin 1 partons with gyromagnetic ratio equal to 1. Such objects are alien to renormalizable quantum field theory, and have nothing in common with our considerations here.

orthogonal neutral (massive) field shall be denoted by X_μ. The mixing angle θ is related to the coupling constants as $\sin\theta = g'/(g^2+g'^2)^{1/2}$. Then the electric charge unit is $e = g'\cos 0 = g\sin\theta$. The mixing is actually small ($\theta \approx 0$) since basically g is the strong coupling while g' is the electromagnetic coupling.

The formalism of such a theory being well-known, we shall immediately proceed to consider its properties.

(A) The leptons are coupled only to A_μ and X_μ. The direct ($\bar{l}l$X) coupling is $g'\sin\theta$, hence is of order e^2. The presence of such an exotic leptonic coupling should not affect the presently known precision tests of QED if the X-particle is moderately massive. (From muon $g-2$ experiments we obtain the limit of $M_X \geqslant$ 2.5 GeV.)

(B) The theory defined by the Lagrangian in eq. (1) is not asymptotically free. However, the ways in which it fails to be asymptotically free may not be essential. First, the gauge group contains an abelian factor. However its coupling constant g' is small and will not become important in the renormalization group until enormous momenta are reached. (In other words, the failure will not manifest itself until such momenta range that higher order electromagnetic processes become important.) Second, the theory contains too many scalar particles whose quartic self-coupling are not driven to zero in the ultraviolet limit [6]. However, if we make the reasonable assumption that these quartic couplings are small then an argument similar to the one above is applicable for those couplings also. Alternatively it may be hoped that introduction of fundamental scalars is unnecessary to achieve spontaneous symmetry breaking [7].

(C) Let us now see what our theory gives us when regarded as a model for short distance processes. With the remarks of (B) in mind, we shall be interested only in the "asymptotic" domain when the effective coupling $\bar{g}$ has become small, while $\bar{g}'$, as well as the effective scalar couplings, have not grown large. In this energy regime we can obtain the standard parton picture within the renormalization group framework, namely parton–parton interactions ($\bar{g}$) can be ignored in the calculation. In the following discussion we shall not explicitly take the Higgs particle into our consideration. These masses may be very large (again consistent with the assumption of small quartic coupling and relatively large vacuum expertation values)[‡2], then they do not contribute in the momentum range we are interested in. In any case the effects of spin 0 partons are well-known and, as we shall see, rather indistinguishable from spin 1 partons of our theory.

e^+e^- *annihilation.* (i) If the masses of the vector gluons are much larger than the momenta under consideration, then we recover the standard Han–Nambu prediction:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 4. \tag{2}$$

(ii) If, on the other hand, the momenta in the problem are greater than all the gluon masses then we must add contributions of spin 1 partons [$R \equiv R_{(1/2)} + R_{(1)}$] and also we have to consider the coherent contributions of diagrams with an intermediate photo and those with an intermediate X. When the produced particles are quark parts, the ratio R[‡3] is modified to read

$$R_{(1/2)} = 2(1+\tan^2\bar{\theta}) = 2(1+g'^2/\bar{g}^2). \tag{3}$$

When the produced particles are gluons, the ratio R depends on the gluon mass ratio

$$R_{(1)} = \tfrac{1}{4}(M_X/M)^4, \tag{4}$$

where M is the mass of the produced gluons[‡4]. In this high energy limit we must also include the contribution of these spin 1 partons to the structure functions for the inclusive process, $e^+e^- \to p$ + anything. A detailed calculation shows that they do not contribute to $\bar{F}_1$, the transverse structure function, in the scaling limit, just as the case for spin 0 parton. The contributions to $\bar{F}_2$ is again proportional to $(M_X/M)^4$, like the case for total annihilation cross section. (iii) For momentum ranges intermediate to those considered in (i) and (ii), even the *qualitative* behavior will be sensitive to the ratio M_X/M. Details will be given in a

‡2 Recall that the vacuum expectation values are roughly proportional to Higgs masses and inversely proportional to quartic couplings.

‡3 We note that the process $e^+e^- \to X \to \mu^+\mu^-$ is of higher order in α.

‡4 For simplicity we have assumed that the two sets of charged gluons are of equal mass $M_1 = M_2 \equiv M$. Generally they are different and the result in eq. (4) should read $R_{(1)} = M_X^4(M_1^{-4} + M_2^{-4})/8$. These results can be easily obtained from the general cross section formulae given by Cabibbo and Gatto [8].

future publication [9].

Electroproduction. The scaling behavior will be again dependent on relative values of q^2 with respect to the gluon masses. For $|q^2| \ll M^2, M_x^2$ our model reduces to the standard (integrally charged) quark–parton model and would reproduce its sum rules[‡5]. For $|q^2| \gtrsim M^2, M_x^2$ scaling will be violated. For $|q^2| \gg M^2, M_z^2$, scaling will be once again recovered but with $F_L(x) \neq 0$. The precise value of F_L/F_T will be rather model dependent just as in the corresponding annihilation situation[‡6]. However, it is safe to say that violations would be largest for small x (large virtual mass of produced particles).

(D) It poses no serious difficulty to graft weak interactions into this theory. The most straightforward way would be a gauge theory based on $SU(3)' \times U(1) \times SU(2)_L$. The mixing problem is slightly more complicated (there will be an additional mixing angle similar to the one in the Weinberg–Salam theory). Details of such a theory will be given elsewhere [9]. We shall only remark that there will be no problem with parity violation in order α. Since the leading terms due to W-boson radiative correction corresponds to the super-asymptotic momentum region (space like $|q^2| \gg M_w^2$), the soft symmetry breaking effects disappears in this limit, leading to the conclusion that order α contribution should commute with the strong $SU(3)'$ group. Then, by Weinberg's argument [11] they can always be rotated away by a redefinition of fermion fields.

‡5 In particular, the Callan–Gross relation holds $F_2 - 2xF_1 = 0$.

‡6 One may speculate that nucleons (or observed hadrons in general) correspond to regions where symmetry is restored in a scheme similar to that discussed by Lee and Wick [10]. In such a "Bubble-in-the-Stream" model of hadrons precocious scaling arises naturally in electroproduction processes, and the fractionally charged quark–parton model is recovered.

In conclusion we think that we have demonstrated that the Han–Nambu model can be put inside the framework of renormalizable field theory and gives a reasonable picture of scaling phenomena (with very distinctive signatures, to be sure). In our opinion the possibility of integrally charged quarks as an alternative to confinement should not be too lightly dismissed.

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References

[1] M.Y. Han and Y. Nambu, Phys. Rev. 139 (1965) B1006.
[2] G. Tarnopolsky et al., Phys. Rev. Lett. 32 (1974) 432; B. Richter, talk presented at XV Int. Conf. on High energy physics, London (1974).
[3] For a review, see for example, J. Ellis, CERN report 1880 (1974).
[4] D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.
[5] J. Clymans and G.J. Komen, CERN report 1844 (1974).
[6] D.J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633; T.P. Cheng, E. Eichten and L.F. Li, Phys. Rev. D9 (1974).
[7] Y. Nambu and G. Jona-Lasino, Phys. Rev. 122 (1961) 345.
[8] N. Cabibbo and R. Gatto, Phys. Rev. 24 (1961) 1577.
[9] T.P. Cheng and F. Wilczek, in preparation. In this paper the properties of the theories presented in this paper will be analyzed using renormalization group methods and the incorporation of weak interactions will be discussed in detail.
[10] T.D. Lee and G.C. Wick, Phys. Rev. D9 (1974) 2291.
[11] S. Weinberg, Phys. Rev. D8 (1973) 4482.
[12] J. Pati and A. Salam, Phys. Rev. D8, 1240, have proposed a very interesting unified gauge theory with integrally charged quarks and gluons.