Comments and Addenda

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Comments on a $V-A$ theory of muon-number nonconservation

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We report the results of our calculations on $\mu\rightarrow ee\bar{e}$, $K_L\rightarrow e\bar{\mu}$ in a gauge theory with three left-handed lepton doublets, which as been proposed recently by other authors in connection with muon-number nonconservation. Other remarks about this model are also included.

In recent communications[1,2,3] we have emphasized that, in the context of modern gauge theories, one would not *a priori* associate a strict conservation law with the muon number. Since it is a lepton flavor number, one would not expect it to be conserved in the weak interactions, just as strangeness is not conserved. More precisely, flavor-number-changing effects in gauge theories are controlled by the off-diagonal terms of the fermion mass matrix in the weak eigenstate space. In the Weinberg-Salam $(V-A)$ theory, the electron and muon are only coupled to the nuetrinos; hence these off-diagonal mass terms, if not identically zero, must be extremely small. However, in theories where muon and electron also couple to heavy leptons with masses in the GeV range, one would have rates for muon-number-violation processes such as $\mu\rightarrow e\gamma$, $\mu\rightarrow ee\bar{e}$, and $K_L\rightarrow e\bar{\mu}$ not extremely small when compared to their present experimental limits. We also note that off-diagonal mass terms are related to the product of mixing angles and mass differences in the physical particle space. Symbolically we may represent this as $(\theta\Delta m^2)$. For example, the strangeness-changing neutral-current effects in the standard model are controlled by $\cos\theta_C\sin\theta_C(m_c^{\,2}-m_u^{\,2})$.

In weak-interaction theories where the right-handed electron and muon belong to nontrivial representations of the gauge group, they must necessarily be coupled to heavy leptons. Thus it is very natural to conjecture that in that class of theories the off-diagonal mass term $(\theta\Delta m^2)$ will be in the GeV^2 range: Namely, we expect the mixing angle θ not to be particularly small and Δm^2 should be of few GeV^2. We have already proposed a model of leptons where, in addition to the standard left-handed doublets, there are two right-handed electron and muon doublets as well. This has the important implication that the electronic neutral current is purely vector; hence parity-violation effects in high-Z atoms should be strongly suppressed.

Very recently a number of authors, Fritzsch,[4] Glashow,[5] Lee, Pakvasa, Shrock, and Sugawara,[6] have independently suggested a $V-A$ model in which $(\theta\Delta m^2)$ is also supposed to be in the GeV^2 range, thus predicting the same size of muon-number-nonconservation effects as our model. In their scheme, they must choose relatively small mixings, and hence a larger value for Δm^2.

Their $V-A$ model is

$$\begin{pmatrix}(1-\theta_e^{\,2}/2)\nu_e'-\frac{1}{2}\theta_e\theta_\mu\,\nu_\mu'-\theta_e N\\ e\end{pmatrix}_L,$$

$$\begin{pmatrix}(1-\theta_\mu^{\,2}/2)\nu_\mu'-\frac{1}{2}\theta_e\theta_\mu\nu_e'-\theta_\mu N\\ \mu\end{pmatrix}_L, \tag{1}$$

$$\begin{pmatrix}N+\theta_e\nu_e'+\theta_\mu\nu_\mu'\\ U\end{pmatrix}_L,$$

where U is identified to be the 1.9-GeV heavy lepton of Perl *et al.*[7]

Clearly the electronic neutral-current structure of this model is identical to that of the Weinberg-Salam model. It predicts substantial parity violation in atomic physics. Hence the viability of this model is predicated upon the possible modifications of the present experimental indications and/or their theoretical interpretations.[8]

While lepton-hadron universality and μe universality[9] are believed to be experimentally checked to the less-than-one-percent level, in this model it is difficult to obtain directly bounds on the individual lepton mixing angles. This is so because there will be mixings among the quark fields, which are also unknown. (For this model it is natural to have six quarks in the three left-handed doublets.) For the $\mu \to e$ processes it is the combination $\theta_e\theta_\mu$ that always appears [see Eqs. (2), (4), and (9)]. Experimental nonobservation of $\nu_\mu + p \to e + X$, etc. can lead one to conclude that $|\theta_e\theta_\mu| < 0.05$.[10] Even arbitrarily taking this maximal value one still would need a fairly large mass for N, $m_N > 10$ GeV, in order to have a $\mu \to e\gamma$ branching ratio[1] in the 10^{-9} range:

$$\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\overline{\nu}_e\nu_\mu)} = (\tfrac{3}{32})(\alpha/\pi)(\theta_e\theta_\mu m_N^2/M_W^2)^2 . \qquad (2)$$

The purpose of this note is to present results of our calculations on $\mu\to ee\overline{e}$ and $K_L \to e\mu$ in this model.

$\mu\to ee\overline{e}$

The ratio $\Gamma(\mu\to ee\overline{e})/\Gamma(\mu\to e\gamma)$ is an important quantity for such models, since it is less sensitive to the parameters of the theory. The authors of Ref. 6 have already computed this ratio by keeping only the $\log(M_W^2/m_N^2)$ factors.[11] With this approximation the calculation is relatively straightforward since the one-photon exchange contribution, which does not have $\log(M_W^2/m_N^2)$ terms, can be dropped completely. However, it is difficult to justify such an approximation in this model. m_N cannot be very small; the log factors must be less than 4. Recently we have carried out a detailed calculation of the corresponding quantity of our $V+A$ model by keeping all the constant terms, in addition to the logs. We can easily carry over our calculations and apply them to this $V-A$ model. We shall only provide the principal intermediate steps and refer the interested readers to Ref. 3 for details.

There are basically three classes of one-loop diagrams contributing to the decay $\mu\to ee\overline{e}$. They are the one-photon exchange, the Z, and $2W$ exchange diagrams. (The latter two will be referred to collectively as the weak amplitude.) The leading amplitude may be parametrized as

$$\begin{aligned}\mathfrak{M}(\mu\to 3e) = {}& \{e\kappa[\overline{u}_e(k_2)\gamma^\lambda v_e(k_3)]/q^2\}\{\overline{u}_e(k_1)(1+\gamma_5)[if_M m_\mu\sigma_{\lambda\nu}q^\nu + f_E(\gamma_\lambda q^2 - \gamma\cdot q q_\lambda)]u_\mu(p)\}\\ &+ e\kappa[\overline{u}_e(k_1)(1+\gamma_5)\gamma_\lambda u_\mu(p)]\{\overline{u}_e(k_2)[g_L(1+\gamma_5)/2 + g_R(1-\gamma_5)/2]\gamma^\lambda v_e(k_3)\} - (k_1 \leftrightarrow k_2) ,\end{aligned} \qquad (3)$$

where

$$\kappa = (e/16\pi^2)(g^2/8M_W^2)(\theta_e\theta_\mu m_N^2/M_W^2) . \qquad (4)$$

Our calculation yields

$$\begin{aligned} f_M &= -\tfrac{1}{2} ,\\ f_E &= -2 ,\\ g_L &= 2\{\ln(m_N^2/M_W^2) + 3 + [(\theta_e^2 - 2)/2\sin^2\theta_W]\} ,\\ g_R &= 2[\ln(m_N^2/M_W^2) + 3] . \end{aligned} \qquad (5)$$

We then obtain the ratio

$$R = \frac{\Gamma(\mu\to ee\overline{e})}{\Gamma(\mu\to e\gamma)} = R^\gamma + R^W + R^{\gamma W} \qquad (6)$$

with

$$\begin{aligned} R^\gamma &= (\alpha/\pi)\{\tfrac{2}{3}[4\ln(m_\mu/2m_e) - \tfrac{13}{6}]f_M^2 - 2f_M f_E + \tfrac{1}{2}f_E^2\} ,\\ R^W &= (\alpha/\pi)(g_R^2 + 2g_L^2)/6 ,\\ R^{\gamma W} &= (\alpha/\pi)(f_E - 2f_M)(g_R + 2g_L)/3 , \end{aligned} \qquad (7)$$

where R^γ, R^W, and $R^{\gamma W}$ stand for the one-photon, weak, and their interference contributions, respectively.

We have calculated the ratio R of Eq. (6) and found $R \simeq 0.06$ to 0.02 for $m_N \simeq 10$ GeV to 30 GeV.

$K_L \to e\mu$

We first calculate the effective Lagrangian from the free-quark box diagram corresponding to the $s\mu\to de$ process. This is then placed between the hadronic state to obtain the amplitude

$$\begin{aligned}\mathfrak{M}(K_L\to e\mu) = {}& (\sin\theta_C G_F/\sqrt{2})\epsilon K[\overline{e}\gamma^\lambda(1-\gamma_5)\mu]\\ &\times\langle 0|(\overline{s}\gamma_5\gamma_\lambda d + \overline{d}\gamma_5\gamma_\lambda s)|K_L\rangle \end{aligned} \qquad (8)$$

with

$$\epsilon = (\alpha/8\pi)(\cos\theta_C/\sin^2\theta_W)\theta_e\theta_\mu , \qquad (9)$$

$$K = (m_c^2/M_W^2)\ln(m_c^2/m_N^2) . \qquad (10)$$

Using the SU_3 relation

$$\langle 0|\overline{s}\gamma_5\gamma_\lambda d + \overline{d}\gamma_5\gamma_\lambda s|K_L\rangle = \sqrt{2}\,\langle 0|\overline{s}\gamma_5\gamma_\lambda u|K^+\rangle \qquad (11)$$

and the branching ratio of $K^+ \to \nu\mu^+$, we obtain

$$\frac{\Gamma(K_L \to e\bar{\mu})}{\Gamma(K_L \to \text{all})} \simeq 5.5\epsilon^2 K^2\,, \tag{12}$$

which ranges from 3 to $7\times10^{-11}(\theta_e\theta_\mu)^2$ for $m_N \simeq 10$ to 30 GeV. This should be adequate for order-of-magnitude estimates even though the effects of strong interactions have not been explicitly taken into account.

Note added. Drs. B. W. Lee and R. E. Shrock have kindly pointed out to us that because of the possible presence of quark mixings, the lepton angles θ_e and θ_μ may not be determined individually from existing experimental bounds on universality. This point was not fully stated in the original version of this manuscript. We are grateful to these authors for this communication. We should also note that W. J. Marciano and A. I. Sanda [Phys. Rev. Lett. 38, 1512 (1977)] have also calculated $\mu \to ee\bar{e}$ in this $V-A$ model.

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[9]In Ref. 5, universality is maintained by taking *all* the lepton and quark mixings to be the Cabibbo angle. However, the author did not present arguments to show that such a restriction can be made "natural."
[10]See, for example, E. Bellotti, D. Cavalli, E. Fiorini, and M. Rollier, Lett. Nuovo Cimento 17, 553 (1976).
[11]S. B. Treiman, F. Wilczek, and A. Zee [Phys. Rev. D 16, 152 (1977)] have earlier calculated this ratio for our $V+A$ model by this leading-log approximation. This is more plausible in that case since the heavy-lepton masses are expected to be much smaller.