Weak-interaction-induced neutrino oscillations

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Neutrino oscillation is a natural theoretical possibility in gauge theories if we do not *a priori* exclude the right-hand components of neutrino fields. The quantitative aspects of the oscillation phenomenon (oscillation length, etc.) are controlled by the neutrino mass matrix. We explore the possibility that neutrinos are massless in the lowest order of weak interactions and higher-order radiative corrections bring about small, but calculable, masses. We have made a systematic search of models which would have such a feature. We find that in one class of models, the neutrino-mass matrix elements would generally be less than 10^{-2} eV. But such theories would contain flavor-changing neutral currents involving charged lepton fields. Another class of models, which can easily be compatible with all known weak-interaction phenomenology, would necessarily involve large gauge groups (e.g., direct products of at least three non-Abelian factors) and a large number of unknown parameters (making it impossible to meaningfully estimate the neutrino masses).

I. INTRODUCTION

Over ten years ago Pontecorvo[1] raised the possibility that there may be oscillations among neutrinos of different flavors[2] $\nu_e \rightarrow \nu_\mu \cdots$, much as the strangeness-oscillation phenomenon in the neutral-kaon system: $K^0 \rightarrow \bar{K}^0$. The electron neutrino and muon neutrino as defined by the familiar weak decays $\pi^+ \rightarrow e^+\nu_e$, $\pi^+ \rightarrow \mu^+\nu_\mu, \cdots$ are assumed not to correspond to states of definite mass. In other words, $\nu_e, \nu_\mu \cdots$ are not all strictly massless and are actually coherent superpositions of different mass eigenstates, each of which will of course have different time evolution properties. This quantum-mechanical phenomenon can manifest itself on the macroscopic level in that the flavor content of a neutrino beam will be time-dependent.[3]

Consider the simplest case of two neutrinos. Let ν_1 and ν_2 be the mass eigenstates with masses m_1 and m_2:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle , \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle . \end{aligned} \tag{1}$$

Thus, for example, an initially pure muon-neutrino beam, even when traveling in vacuum, will spontaneously generate electron neutrinos. The probability of finding ν_e at time t is clearly

$$\begin{aligned} |\langle \nu_e|\nu_\mu(t)\rangle|^2 &= |\sin\theta\cos\theta(e^{-iE_1t} - e^{-iE_2t})|^2 \\ &= 2\sin^2\theta\cos^2\theta[1-\cos(E_1-E_2)t] , \end{aligned} \tag{2}$$

where $E_i = (p^2+m_i^2)^{1/2}$ with $p = |\vec{p}|$ being the neutrino momentum. The "oscillation length" (or "oscillation frequency") is then

$$l_{osc} = 2\pi(E_1-E_2)^{-1} \simeq 4\pi p/(m_1^2-m_2^2) . \tag{3}$$

Neutrino oscillation can occur if (i) lepton flavors (muon number, electron number, etc.) are not separately conserved, and (ii) neutrinos are not degenerate in mass; namely in Eqs. (2) and (3) $\theta \neq 0$ and $m_1 \neq m_2$.

These conditions may be stated directly in terms of the flavor space parameters. The neutrino mass matrix defined with respect to ν_e and ν_μ states is not diagonal:

$$(\bar{\nu}_a)_R\, m_\nu^{ab} (\nu_b)_L ; \tag{4}$$

a,b are "flavor indices," namely

$$(\nu_a)_L = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_L , \text{ etc.}$$

with

$$m_\nu^{ab} = \begin{pmatrix} m_\nu^{ee} & m_\nu^{e\mu} \\ m_\nu^{\mu e} & m_\nu^{\mu\mu} \end{pmatrix} . \tag{5}$$

In general

$$m_\nu^{e\mu} \neq m_\nu^{\mu e} . \tag{6}$$

Hence we need a different rotation matrix U, V acting on the left-hand and right-hand fields to diagonalize the neutrino mass matrix[4]

$$V^+ \mathfrak{M}_\nu U = \mathfrak{M} \tag{7}$$

with

$$\mathfrak{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} . \tag{8}$$

Or, equivalently,

$$\mathfrak{M}_\nu^+ \mathfrak{M}_\nu = V \mathfrak{M}^2 V^+ \qquad (9)$$

and

$$\mathfrak{M}_\nu \mathfrak{M}_\nu^+ = U \mathfrak{M}^2 U^+ , \qquad (10)$$

where

$$(\nu_a)_R = V_{ai}(\nu_i)_R \qquad (11)$$

and

$$(\nu_a)_L = U_{ai}(\nu_i)_L ; \qquad (12)$$

i (=1, 2) is the mass-eigenstate index. Equation (12) is of course just our Eq. (1). Comparing the off-diagonal terms on both sides of Eq. (12) yields, for example,

$$m_\nu^{ee} m_\nu^{\mu e} + m_\nu^{e\mu} m_\nu^{\mu\mu} = \sin\theta \cos\theta (m_1{}^2 - m_2{}^2) . \qquad (13)$$

Thus failure to fulfill either of the conditions (i) $\theta \neq 0$ or (ii) $m_1 \neq m_2$ would imply that the matrix $\mathfrak{M}_\nu^+ \mathfrak{M}_\nu$ is diagonal. In fact we can symbolically represent the oscillation phenomenon by the "mass-insertion" diagram shown in Fig. 1. In this sense we can regard the matrix elements of m_ν^{ab} as the "coupling constants" of the oscillation process. In the following discussion the oscillation conditions will be stated directly in terms of these flavor-space parameters, and by "neutrino mass" we shall always mean the matrix m_ν in Eq. (5).

In previous communications[5] we have reported our investigations on the question of muon-number nonconservation in weak interactions. Our point was the simple observation that in gauge theories renormalizability requires the most general gauge-invariant couplings between scalar and fermion multiplets. Spontaneous symmetry breaking generally brings about fermion masses that are not diagonal with respect to the weak eigenstates. Thus, in gauge theories, we would not expect (quark and lepton) flavor conservation, unless dynamics brings about accidental symmetries. In particular, we would expect neutrino oscillation if neutrinos are not massless.

In most guage theories as typified by the Weinberg-Salam model,[6] the right-handed components of neutrino fields are assumed to be absent. In this way we are guaranteed to have massless particles and the absence of ν oscillation. In terms of Fig. 1 it is clear that there cannot be a $(\nu_\mu)_L \to (\nu_e)_L$ transition if all the intermediate ν_R's are absent. Thus, had it not been for our *a priori* banishment of the right-handed neutrino fields, oscillations among different-flavor neutrino states would have indeed been a natural gauge-theoretical possibility. In other words, while the two conditions (i) and (ii) for neutrino oscillations are logically independent, in gauge theories, once we have massive neutrinos it is very plausible to expect oscillations to occur.

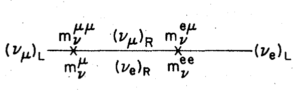

FIG. 1. Symbolic representation of $\nu_e \rightleftarrows \nu_\mu$ oscillation in terms of the neutrino-mass matrix.

We may also remark here that in certain gauge models, because of mixings among neutral leptons, the neutrino flavor states as defined in $\pi \to (\mu\nu_\mu)$ and $(e\nu_e)$ decays are not orthogonal.[7] Thus there can be a violation of the classic "two-neutrino" experiments,[8] namely, the sequence

$$\begin{array}{l} \pi^+ \to \mu^+ \nu_\mu \\ \qquad\quad \downarrow \\ \qquad\quad \nu_\mu + n \to p + e \end{array}$$

can take place. But we must emphasize that this phenomenon is not neutrino oscillation. The probability of this reaction has no time dependence which is characteristic of an oscillation. It can take place even for strictly massless neutrinos. In this paper we shall not discuss this phenomenon of neutrino nonorthogonality.

While neutrino oscillation is a natural theoretical possibility in gauge theories, the question remains whether it is possible for us to make some quantitative theoretical statement. The relevant parameters are clearly the mixing angle and neutrino mass differences in Eqs. (2) and (3). More directly, we are interested in the neutrino-mass matrix elements m_ν^{ab} in Eq. (5); as is well known mass parameters are generally not calculable in quantum field theory. However, neutrinos, if not strictly massless, must be extremely light when viewed on an ordinary mass scale. It is perhaps plausible then to explore the following possibility: Neutrinos are massless in the lowest order of weak interactions and higher-order radiative corrections bring about their masses. This will be the basic premise we make in this work. Our program will be to search for the simplest gauge models where right-hand neutrino fields are present, but spontaneous symmetry works in such a way that neutrinos stay massless (as a natural zeroth-order relation) and the matrix elements m_ν^{ab} in Eq. (5) are given by radiative correction diagrams as shown in Fig. 2.[9]

In field theories, higher-order contributions to particle masses are generally divergent and we need (infinite) counterterms to cancel such contributions, leaving masses as free parameters of the theory. However, in gauge theories this problem may be circumvented under certain special circumstances. If the neutrino-mass counterterms

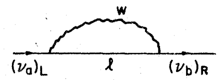

FIG. 2. General second-order radiative-correction diagram for the neutrino mass.

are naturally forbidden, say on grounds of gauge invariance (here the word naturality is used in the technical sense that the desired zeroth-order relation is invariant under arbitrary changes of parameters over some nonvanishingly small range), higher-order contributions cannot be divergent since the necessary counterterms are not available to cancel the infinities. Some details of the calculations will be provided in Sec. II. We only mention that finiteness of a diagram such as Fig. 2 in such models comes about because of a cancellation mechanism similar to that first invented by Glashow, Iliopoulos, and Maiani (GIM) in connection with strangeness-changing neutral-current suppressions.[10] We note that for a class of models (listed as class A in Sec. II) it involves cancellation among diagrams with different intermediate fermions. This is similar to the more familiar GIM situation. However, in class B models, the Fig. 2 loop contribution becomes finite because of a cancellation among diagrams with different intermediate vector-boson lines.

We have examined many models in these two classes, and a few simple and representative examples will be reported below. Although we cannot claim to have made an exhaustive search (partly because we are not interested in gauge models with very complicated structures), our general conclusion is not a very encouraging one. In class A models (with GIM on fermion lines) it is difficult to avoid flavor-changing neutral currents involving charged leptons. There is no natural mechanism for excluding a $\mu e Z$ coupling in the lowest order. This is a very undesirable feature in view of the fact that processes such as $\mu \to 3e$ are known to be extremely suppressed.[11] It is not too difficult to have class B models that are phenomenologically satisfactory. However, these models necessarily involve large groups (a product of three non-Abelian groups is needed) because of the extended GIM mechanism on boson lines.

In class A models the one-loop contribution is automatically small; it is of the order of $(\alpha/\pi)(m^3/M_W^2)$. m is some lepton mass and M_W is the mass of the weak intermediate boson. On the other hand, the extended-GIM-on-boson mechanism in class B models only brings about logarithmic suppression. The results may nevertheless be sufficiently small because of the presence of mixing angles. But the presence of these multitudes of extra parameters makes it difficult to have even a rough order-of-magnitude estimate of the quantities we want to compute.

II. CALCULABILITY OF NEUTRINO MASSES IN GAUGE THEORIES

Here we shall present a simple discussion on the subject of neutrino mass being *naturally* zero at the tree graph level, and hence higher-order weak contributions will be finite and "calculable." For a related discussion the reader is referred to the paper by Georgi and Glashow,[12] who first presented results of electron-muon mass-ratio calculations based on the assumption that the electron mass results entirely from radiative corrections.

The weak-interaction models we shall discuss will be divided into two classes depending on the ways the absence of the zeroth-order neutrino-mass term $\bar{\nu}\nu$ comes about (and it is correlated to the ways the GIM cancellation mechanism functions in making Fig. 2 finite).

Class A. Zeroth-order neutrino-mass terms $\bar{\nu}\nu$ are absent because there are no appropriate Higgs scalars for them to couple to.

Class B. The Yukawa couplings between neutrino fields and scalar fields ($\bar{\nu}\nu\phi$) *do* exist. However, the particular Higgs scalar naturally does not develop a vacuum expectation value (VEV).

Consider the prototype self-energy diagram of Fig. 3 with initial and final neutrino states ν_i and ν_j and an intermediate gauge boson w_λ^σ (mass M_σ) and fermion l (mass m_l):

$$\Sigma_{ij}(p^2) = \int \frac{d^4k}{(2\pi)^4}\left(\frac{it_L^\sigma}{2\sqrt{2}}\right)^+_{jl} \gamma_\alpha(1+\gamma_5)\frac{i}{\not{p}+\not{k}-m_l} \times \left(\frac{it_R^\sigma}{2\sqrt{2}}\right)_{li} \gamma_\beta(1-\gamma_5)\left(\frac{-ig^{\alpha\beta}}{k^2-M_\sigma{}^2}\right), \quad (14)$$

where $t_{L,R}^\sigma$ are the coupling matrices for $\bar{\nu}\gamma_\lambda(1 \mp\gamma_5)lw_\sigma^\lambda$.

Using the dimensional-regularization procedure and expanding the result around the physical dimension of $n=4$, we have

$$\Sigma(0) = i(16\pi^2)^{-1}(t_L^{\sigma+} m_l t_R^\sigma)(1-\gamma_5) \times [C + I(m_l{}^2, M_\sigma^2)] \quad (15)$$

with

$$I(m^2,M^2) = \frac{m^2\ln m^2 - M^2 \ln M^2}{m^2 - M^2} \simeq \ln M^2 - \frac{m^2}{M^2}\ln\frac{m^2}{M^2} + O\left(\frac{m^2}{M^2}\right) \quad (16)$$

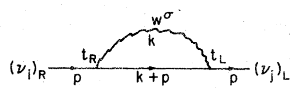

FIG. 3. Prototype second-order self-energy diagram for the neutrino.

and

$$C = \frac{-2}{n-4} + 1 + \Gamma'(1)\,. \tag{17}$$

We are, of course, interested in theories where $\Sigma(p^2)$ is finite; namely, there would be several diagrams like Fig. 3 and in their sum the divergent constant C in Eqs. (15) and (17) cancels. In general, such a GIM cancellation can be seen most transparently when particles are labeled by weak eigenstates. We shall often transcribe the above result in that language in the following discussion.

As we have already stated, if $\bar{\nu}\nu$'s are naturally absent, higher-order contributions must be finite and hence neutrino masses m_ν^{ab} will be "calculable." For class A models it is relatively straightforward to see how this comes about. The leading divergent term $\sim C m_l$ in Eq. (15) is given by the one-mass-insertion diagram of Fig. 4. However, the l's must be in the same weak isomultiplets as the ν's (in order to have the $\nu l W$ couplings). Hence the absence of a $\bar{\nu}_L \nu_R$ term (because of the representation content of the Higgs scalars) would automatically imply the absence of an $\bar{l}_L l_R$ term. Thus $m_l = 0$ and the leading logarithmic divergence vanishes. Explicitly, from Eqs. (15) and (17)

$$\Sigma_{ij}^{\text{div}}(0) \sim C(t_L^{\sigma +} m_l t_R^{\sigma})_{ij} = 0 \text{ for each } \sigma\,. \tag{18}$$

The next leading contribution comes from the "three-mass-insertion" diagram of Fig. 5. This diagram generally survives because the s's do not have to be in the same representation as l and ν; the terms $\bar{l}s$, $\bar{s}s$, etc. need not be absent. From Eqs. (15), (16), and (18)

$$\Sigma_{ij}(0) = -i(16\pi^2)^{-1} t_L^{\sigma +} m_l^3 t_R^{\sigma} M_\sigma^{-2} \ln(m_l^2/M_\sigma^2)\,. \tag{19}$$

In Eq. (14) we have taken the 't Hooft gauge propagator for the intermediate vector boson. There are then also contributions coming from the corresponding diagrams in which the gauge boson of Fig. 3 is replaced by the unphysical Higgs particle. Again the logarithmic-divergent term would have come from the one-mass-insertion diagram of Fig. 4(b). This graph is absent basically for the same reason that Fig. 4(a) is absent: They correspond to $\bar{\nu}\nu\phi$ counterterms in the symmetric theory and are not present in class A models. The leading Higgs contribution that survives is Fig. 5(b): In a self-consistent class A model $\bar{\nu}_R S_L$ and $\bar{\nu}_L S_R$ must couple to *different* sets of Higgs multiplets ϕ_1, ϕ_2. After spontaneous symmetry breaking ϕ_1 and ϕ_2 mix. In short, there is a GIM cancellation mechanism among different Higgs graphs. Clearly the leading contribution is again of the order of m_l^3/M_σ^2 (up to finite logarithms) and thus does not dominate over the gauge boson contribution. Since we are only interested in order-of-magnitude estimates of these self-energy diagrams, we shall not bother to display the Higgs contribution separately.

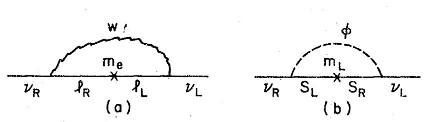

FIG. 4. Second-order self-energy diagrams in terms of the weak eigenstates with one mass insertion on the fermion line.

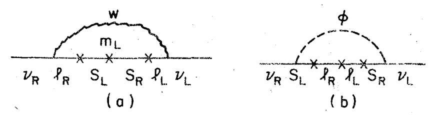

FIG. 5. Second-order self-energy diagrams in terms of the weak eigenstates with three mass insertions on the fermion line.

The situation for class B models is slightly more complicated. Here the masslessness of the neutrinos is originally arranged by letting the appropriate vacuum expectation value be naturally zero. True naturality would then guarantee that higher-order corrections to VEV are finite also. Consider the diagram in Fig. 4(a). For class B models the one mass insertion on the intermediate fermion line generally does not vanish (since this would imply that the entire multiplet of Higgs cannot develop a VEV). Hence, one would need some other cancellation mechanism than a GIM cancellation mechanism on the fermion line. We can try the GIM cancellation mechanism on the intermediate vector-boson line by one mass insertion. Namely, the W at the left-hand ν vertex is different from W at the right-hand ν vertex ($W_L \neq W_R$). W_L and W_R can then mix through an off-diagonal gauge boson mass term. More explicitly, we consider

$$\mathcal{L}_W = \bar{\nu}_L \gamma_\alpha T_L^A l_L W_L^{A\alpha} + \bar{\nu}_R \gamma_\alpha T_R^B l_R W_R^{B\alpha}\,, \tag{20}$$

where T_L^A and T_R^B are the coupling matrices and A and B are the left-hand and right-hand indices, respectively. The gauge-boson mass eigenstates are then the linear superposition of W_L and W_R,

$$W^\sigma = X^{\sigma A} W_L^A + X^{\sigma B} W_R^B\,, \tag{21}$$

where X is the appropriate rotation matrix and $X^{\sigma A}, X^{\sigma B}$ are the parts which act on left-hand and right-hand fields, respectively. The self-energy

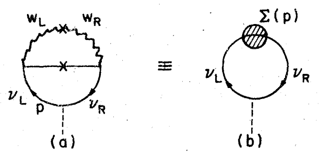

FIG. 6. Tadpole diagram for the Higgs scalar which couples to the neutrino.

contribution in Eq. (15) is now modified to read

$$\Sigma(0)=i(16\pi^2)^{-1}(1-\gamma_5)\sum_\sigma (T_L^{A+}m_l T_R^{B})(X^+)^{A\sigma}(X)^{\sigma B} \times[C+I(m_l^2,m_\sigma^2)]\,. \quad (22)$$

By the orthogonality condition of X

$$(X^+X)^{AB}=0\,, \quad (23)$$

we obtain

$$\Sigma(0)=i(16\pi^2)^{-1}(1-\gamma_5)\sum_\sigma (T_L^{A}m_l T_R^{B})(X^+)^{A\sigma} \times(X)^{\sigma B}\ln M_\sigma^2\,. \quad (24)$$

Thus Fig. 2 will be finite. Yet we still do not have a self-consistent solution at this stage. We can see this by examining the tadpole diagram of Fig. 6. By simple power counting we have $\Sigma(p^2)\sim p^{-2}$. Consequently this contribution to the vacuum expectation value $\langle\phi\rangle\sim\int d^4p/p^4$ is divergent. This implies the need for an appropriate counterterm in the scalar potential, and this would make $\langle\phi\rangle$, and thus the neutrino masses, free parameters of the theory. In other words, $\langle\phi\rangle=0$ is not a "natural" zeroth-order relation after all. Thus generally, we need further suppressing power so that tadpole diagrams can be finite also. This leads us to the self-energy diagram of Fig. 7. Their contribution to the neutrino mass is given also by Eqs. (22) to (24). But their contribution to $\langle\phi\rangle$ being proportional to $\int d^4p/p^6$ is finite. This implies that class B models would generally involve a direct product of three simple gauge groups, $G_L\times G_R\times G_S$, in order to achieve the desired suppression power for the GIM cancellation mechanism on vector bosons.

III. EXAMPLES OF GAUGE MODELS WITH CALCULABLE NEUTRINO MASSES

Our aim here is not to make an exhaustive search of all possible models which have a finite

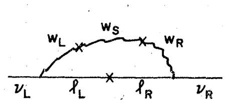

FIG. 7. Second-order self-energy diagram in terms of the weak eigenstates with two mass insertions on the boson line and one mass insertion on the fermion line.

and calculable neutrino mass matrix. We shall attempt to construct "simple" models which are self-consistent and are compatible with known weak-interaction phenomenology for leptons. By the criterion of "simplicity" we mean that we shall by and large restrict ourselves to product groups of SU_2 (and U_1).[13] Within these groups we shall also confine ourselves to the smallest possible representations.

Class A models

Here we discuss models in which the $\bar\nu\nu$ terms are naturally absent because neutrinos are forbidden to couple to any Higgs scalars.

First, consider the following simple example: The group is $SU_2\times U_1$. We have the lepton doublets $(y=-1)$

$$(\psi_a)_{L,R}=\begin{pmatrix}\nu_a\\ l_a\end{pmatrix}_{L,R}$$

and singlets $(y=-2)(s_a)_{L,R}$, where $a=1,2$ and l and s are negatively charged leptons. The absence of bare mass terms $\bar\psi_a\psi_b$ for the doublets can only be enforced by a set of *ad hoc* discrete symmetries, while the bare mass terms $\bar s_a s_b$ for the singlets are allowed. A set of doublets of Higgs scalars $(y=+1)$

$$\Phi_i=\begin{pmatrix}\phi_i^+\\ \phi_i^0\end{pmatrix}$$

can then couple to leptons as $\bar\psi_a\tilde\phi_i s_b$ and yield mass terms $m_{ab}\bar l_a s_b$ when ϕ develops a VEV. Because of the discrete symmetry, the left-hand doublet ψ_L must couple to a different Higgs doublet from that of the right-hand doublet ψ_R. Clearly $\{l_a\}$, $\{s_a\}$ are weak eigenstates. They are linear combinations of e, μ, E, M which diagonalize the mass matrix. But when examining the one-loop radiative contribution to the neutrino masses, it is still more convenient to label the intermediate fermions by the weak eigenstate. It is then immediately clear that the absence of a $\bar\nu_a\nu_b$ term in zeroth order also implies the absence of an $\bar l_a l_b$ term. Hence the leading logarithmic-divergent one-mass-insertion contribution vanishes as discussed in Sec. II. We have

$$m_\nu^{ab}\simeq g^2(16\pi^2M^2)^{-1}\Big(\sum_{cd} m_{ls}^{ac}m_{ss}^{cd}m_{sl}^{db}\Big) \times\ln(M^2/m^2)\,; \quad (25)$$

m_{ls}^{ac} is the mass term $\bar l_a s_c$, etc. m denotes the average charged lepton mass and m the intermediate-vector-boson mass.

The most serious defect of this model is the presence of flavor-changing neutral currents involving the charged leptons e, μ, E, M.... In particular, we do not have a natural mechanism to eliminate the μeZ coupling. Thus the process $\mu \to ee\bar{e}$ can proceed at the tree graph level and it can be suppressed only by tuning small mixing angles. The smallness of the experimental upper limit for the branching ratio $R_{3e} = \Gamma(\mu \to ee\bar{e})/\Gamma(\mu \to e\nu\bar{\nu})$, of order 10^{-9}, will then limit the ν mass to rather small values. To get a very rough order-of-magnitude estimate, we can take the combination of the mixing angles, which comes in the amplitude for R_{3e}, to be less than 10^{-5}. If we then take the mixing-angle combination in Eq. (25), $(m_{es}m_{ss}m_{se}/m^3)$, to be of the same order of magnitude, we get

$$m_\nu \lesssim \left(\frac{\alpha}{4\pi} 10^{-5} \frac{m^2}{M^2} \ln\frac{M^2}{m^2}\right) m \,. \tag{26}$$

So elements of the neutrino mass matrix would typically be of order of 10^{-2} eV for $m \sim 1$ GeV and $M \sim 60$ GeV.

The μeZ problem is a general feature in this class of models. The basic mechanism of the weak-interaction-induced neutrino mass is that of Fig. 5(a), and ν_L is known to couple (at least predominantly) to the electron and muon, and in order to have mixing the intermediate leptons s must be in representations of a different dimension from that for l's. In other words, the charged lepton must necessarily have different weak isospin and this brings about the μeZ coupling.[14]

Another potential problem is the presence of an electron axial-vector neutral current. This can contribute to the leading (coherent) parity-violation effects in heavy atoms.[15] Then there is of course the unattractive feature that we have had to impose discrete symmetry on the model in order to get rid of the $\bar{l}_a l_b$ bare mass term. This discrete symmetry, however, can be removed by extending the model to a larger group such as $SU(2)\times U(1)\times U(1)$ or $SU(2)\times SU(2)\times U(1)$.

Class B models

As the discussion in Sec. II has indicated, to construct a class B model one needs the product of (at least) three non-Abelian groups. Here we illustrate this with an $SU(2)_L \times SU(2)_R \times SU(2)_S \times U(1)$ model. The doublet leptons are ($a = 1, 2, 3$)

$$\begin{aligned}(\psi_a)_L &= \begin{pmatrix} n_a \\ l_a \end{pmatrix}_L \sim (2,1,1), \quad y = -1\,,\\ (\psi_a)_R &= \begin{pmatrix} n_a \\ l_a \end{pmatrix}_R \sim (1,2,1), \quad y = -1\,,\end{aligned} \tag{27}$$

and there is also a singlet neutral lepton

$$(n_4)_{L,R} \sim (1,1,1)\,, \quad y = 0\,.$$

There is a Higgs multiplet $\phi \sim (2,2,1)$, $y = 0$ which couples to $\bar{\psi}_L \phi \psi_R$. We can arrange the Higgs potential in such a way that

$$\langle\phi\rangle = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

but with $\lambda_1 = 0$. In this way all the charged leptons l_a pick up mass (in fact we can choose l_a to be the mass eigenstates e, μ, τ) but the corresponding neutral leptons stay massless. There are also two sets of Higgs doublets $\phi_1 \sim (2,1,1)$, $\phi_2 \sim (1,2,1)$ with $y = 1$. This way $\langle\phi_1\rangle$ and $\langle\phi_2\rangle$ give rise to neutral lepton masses m_{4a} and m_{a4}. Thus, if we further impose a discrete symmetry such that $m_{44=0}$ we will have for neutral leptons

$$\bar{n} m^{\text{neutral}} n$$

with

$$m^{\text{neutral}} = \begin{bmatrix} 0 & 0 & 0 & m_{14} \\ 0 & 0 & 0 & m_{24} \\ 0 & 0 & 0 & m_{34} \\ m_{41} & m_{42} & m_{43} & 0 \end{bmatrix} . \tag{28}$$

This form of mass matrix would imply that there are two massless neutrals in the zeroth order (the neutrinos ν_e, ν_μ) and two massive neutral leptons.[16] $\langle\phi\rangle, \langle\phi_1\rangle\langle\phi_2\rangle$ of course also contribute to the intermediate-vector-boson masses. In order to mix W^L and W^S, as well as W^S and W^R, we also need Higgs fields $\chi_1 \sim (2,1,2)$ and $\chi_2 \sim (1,2,2)$. This yields the charged-vector-boson mass matrix as

$$M_{\sigma\rho} = \begin{bmatrix} M_1 & 0 & \delta m_1 \\ 0 & M_2 & \delta m_2 \\ \delta m_1 & \delta m_2 & M_3 \end{bmatrix}, \quad \sigma, \rho = L, R, S\,.$$

First we note that M_{LR} and M_{RL} masses are absent so that at least two mass insertions are needed to join the $W_L^{(+)}$ and $W_R^{(+)}$. We have also for simplicity assumed that the vacuum expectation values are such that the off-diagonal entries are relatively small, $M_{1,2,3} \gg \delta m_{1,2}$. Consequently, $W_{L,R,S}$ are close to their mass eigenstates $W_{1,2,3}$ with masses in the neighborhood of $M_{1,2,3}$. The rotation matrix X, which diagonalizes the charged-W mass matrix,

$$W_\sigma^{(+)} = X_{\sigma A} W_A^{(+)}\,, \quad \sigma = 1,2,3\,, \quad A = L, R, S$$

has small nondiagonal entries.

Thus we have an example which fulfills the requirement of a class B model and the induced neutrino masses are given by Eq. (24). Displaying only the dominant term which is proportional to the heavy-lepton mass m_τ we have

$$m_\nu^{ab} \simeq \frac{\alpha}{4\pi} m_\tau \frac{\delta m_1 \delta m_2}{M_1 - M_2} \theta^L_{\tau a} \theta^R_{\tau b} \times \sum_{i=1,2} \frac{\ln M_i^2/M_3^2}{M_i - M_3},$$

where $\theta^L_{\tau a}$, $\theta^R_{\tau b}$ are the small mixing angles [determined by the mass terms in Eq. (28)] of ν_a and ν_b in the left-hand and right-hand τ doublets. Clearly there are so many unknown parameters that it would be senseless to attempt any numerical estimate.

For the neutral currents of this model we make the obvious observation that they do not change the flavor of charged leptons — all l's are $t_3 = \frac{1}{2}$ members of a doublet. It also can be shown that with the imposition of left-right symmetry $\langle \chi_1 \rangle = \langle \chi_2 \rangle$, etc., the mass eigenstates for neutral vector bosons are

$$A = \frac{\sqrt{2}}{g} Z_v + \frac{1}{g_s} W_3^s + \frac{1}{2g'} B,$$

$$Z_1 = \frac{1}{(g^2/2 + 4g'^2)^{1/2}} [g(W_3^L + W_3^R)/\sqrt{2} + 2g'B],$$

$$Z_2 = \frac{1}{(g^2/2 + 4g^2)^{1/2}} [-g(W_3^L + W_3^R)/\sqrt{2} - g_3 W_3^s],$$

$$Z_A = (W_3^R - W_3^L)/\sqrt{2}.$$

It is then obvious that $Z_{1,2}$ couples only to vector currents and Z_A to axial-vector currents, respectively. In this way regardless of the quark couplings, the parity violation in high-Z atoms will be suppressed. This is compatible with present experimental indications. (Of course, we can still have unequal neutrino- and antineutrino-hadron scattering cross sections simply because the laboratory neutrinos are left-handed— this feature differs from that in the "vectorlike models.")

IV. CONCLUDING REMARKS

In this paper we have made a number of observations about neutrino oscillations in the context of gauge theories of weak interactions. We emphasize that such a phenomenon is a natural theoretical possibility in any theory where the right-handed neutrino fields are not *a priori* excluded from the theory.

The principal purpose of this work was to explore the possibility that neutrino oscillations are induced by the weak interaction. In particular, the extremely small neutrino masses are entirely brought about by their weak interactions. Technically, this means the construction of gauge models in which neutrinos are naturally massless at zeroth order, and higher-order weak radiative corrections endow them with small masses, both the diagonal and off-diagonal type. We have emphasized in particular that the cancellation mechanism operating here on fermion and boson lines to render self-energy diagrams finite is analogous to the famous GIM mechanism. We have reported on the result of our search of simple realistic models that will have these desirable features.

We have constructed an $SU_2 \times U_1$ (or $SU_2 \times SU_2 \times U_1$) model in which the magnitudes of the induced neutrino masses are related to the smallness of the $\mu e Z$ coupling. We have also found one model based on $SU_2 \times SU_2 \times SU_2 \times U_1$ which is not phenomenologically unsatisfactory. However, the results for neutrino masses involve so many unknown parameters that it is difficult to make any reliable estimates about the quantitative aspects of neutrino oscillations. It is hoped that our efforts will be useful for future discussions on this phenomenon in the context of modern gauge theories.

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coupled to electrons can lead to an unacceptably large rate for neutrinoless double-beta decay. [See A. Halprin, P. Minkowski, H. Primakoff and S. P. Rosen, Phys. Rev. D 13, 2567 (1976)]. This problem can be avoided only if particles of the same charge and helicity do not mix—contrary to the spirit of this work.

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[16]One can see this result by simply observing that the matrix in Eq. (28) is really the direct product of two independent 3×1 mass matrices, each of which clearly has one eigenstate with nonzero mass eigenvalues.