Simple model of fourth-generation fermions

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If the mass ratios of the charged leptons, the charge $-\frac{1}{3}$ and $\frac{2}{3}$ quarks are roughly the same in each generation (except the first one), the present experimental limits on the fourth lepton mass and on the deviation of the electroweak ρ parameter from its tree-level value will lead us to expect that such fourth-generation fermions, if they exist, will have mass values in a narrow range. The phenomenological implications of this simple model including the production and decay properties of the seventh (charge $-\frac{1}{3}$) quark with mass ≈ 60 GeV and the contribution by the eighth (charge $\frac{2}{3}$) quark with mass ≈ 450 GeV to higher-order effects such as the kaon CP impurity parameter ϵ are discussed.

I. INTRODUCTION

One of the outstanding questions in the standard model of particle interactions is the fermion replication problem. For example, we still do not have a good theoretical rationale for the number of fermion generations. There are definitely three generations of leptons and quarks with identical gauge coupling. Many people regard the existence of a sequential fourth generation as a real possibility—especially given that the standard cosmological considerations seem to allow for one more flavor of light neutrinos. If the fourth generation exists, questions naturally arise regarding the following. What are the most likely values for their masses and mixing angles? What is the prospect of detecting them in the near future? Are we already seeing their effect in higher-order processes that are sensitive to a superheavy quark contribution in their intermediate states?[1]

In the preceding paper[2] we have demonstrated that the observed systematics of quark masses and mixing angles can be neatly accounted for by the quark mass matrices having the Fritzsch texture and by having the charge $\frac{2}{3}$ and $-\frac{1}{3}$ quark matrices to be closely proportional to each other. This proportionality is broken mainly in the light-quark sector. Namely, except for the first generation, the ratio of the up and down quarks should be, to a good approximation, the same in each generation:

$$R = m_{(2/3)}/m_{(-1/3)} = \text{generation independent}\ . \tag{1}$$

Our analytic approach was facilitated by a power-series expansion of both the quark masses and mixing angles in powers of the Cabibbo angle $\lambda = 0.225$. In this way we can explain the hierarchical pattern of the Kobayashi-Maskawa (KM) angles: $\delta = O(1)$, $\theta_1 = O(\lambda)$, $\theta_2 = O(\lambda^2)$, and also predict $\theta_3 = O(\lambda^3)$.

Since this simple model of quark mass matrices naturally suggests its continuance to the four-generation case, we shall in this paper work out the details of this possible extension. The fourth-generation leptons will be denoted by κ and ν_κ; the seventh (charge $-\frac{1}{3}$) and the eighth (charge $\frac{2}{3}$) quarks by h and o, for hepta- and octa-quarks, respectively:

$$\begin{pmatrix} \nu_\kappa \\ \kappa \end{pmatrix}_L, \quad \begin{pmatrix} o \\ h \end{pmatrix}_L .$$

In Sec. II the likely range of fourth-generation masses is obtained, mainly by using the electroweak ρ-parameter constraint. In Sec. III the fourth-generation KM angles are deduced. The phenomenological implications of such fourth-generation fermions are then studied in the remaining two sections. In Sec. IV we calculate the lifetime and decay properties of hepta-quark with $m_h \simeq 50-70$ GeV. In Sec. V we study the effects of octa-quark with $m_o \simeq 450$ GeV on the CP-violation parameters and on the possibility of a detectable flavor-changing decay $Z \to b\bar{h}$.

II. FOURTH-GENERATION MASSES

We shall assume that ν_κ, like all other known neutrinos, is very light, perhaps massless. For the charged lepton κ we already have an experimental lower bound[3] of

$$m_\kappa > 22 \text{ GeV}\ . \tag{2}$$

Since many grand-unified-theory (GUT) models would have the mass ratio of the down quark to the charged lepton to be 2.5 to 3 [recall the successful relation of $m_b/m_\tau \simeq 2.94$ (Ref. 4)], we expect that the charge $-\frac{1}{3}$ hepta-quark to have a mass

$$m_h > 55 \text{ GeV}\ . \tag{3}$$

Finally we can use the basic result of Eq. (1) to obtain a lower bound for the octa-quark as well:

$$m_o > 420 \text{ GeV} \tag{4}$$

for $R \simeq m_c/m_s = 1.35/0.175 = 7.7$. [Needless to say there is some uncertainty in our knowledge of this ratio R—reflecting mainly the allowed range of $150 \le m_s \le 200$ MeV (Ref. 5).] We should note that these lower bounds

(2)–(4) are consistent with the hierarchical pattern as established by the first three generations: the masses increase by at least an order of $1/\lambda^2$ (i.e., a factor of 10–30) for each succeeding generation.

If the fourth-generation fermions exist their masses cannot be significantly higher than the lower bounds as shown in Eqs. (2)–(4) because of the electroweak ρ-parameter constraint. The low-energy neutrino and antineutrino neutral-current data are found[6] to be consistent with the tree-level relation

$$\rho = M_W{}^2/M_Z{}^2\cos^2\theta_W = 1 \tag{5}$$

with

$$\rho_{\text{expt}} = 1.02 \pm 0.02 \ . \tag{6}$$

On the other hand, each nondegenerate doublet of fermions ($m_1 \neq m_2$) makes a contribution,[7] through the loop diagram, of

$$\Delta\rho(m_1,m_2) = (G_F/8\sqrt{2}\pi^2)\left[m_1{}^2+m_2{}^2-\frac{2m_1{}^2m_2{}^2}{m_1{}^2-m_2{}^2}\ln\frac{m_1{}^2}{m_2{}^2}\right] . \tag{7}$$

Thus, while the corrections due to the first three generations of leptons and quarks (as well as possibly the fourth-generation leptons) are negligible, this will not be the case for the doublet of the highly nondegenerate superheavy seventh and eighth quarks. Indeed, taking the three colors into account,

$$\Delta\rho(m_o = 420, m_h = 55) = 0.049 \ , \tag{8}$$

which already exceeds the one-standard deviation bound of Eq. (6). We do not regard this as definitive evidence against the existence of a sequential fourth generation satisfying the condition of (1) for two reasons: It is quite possible that the ration R of (1) is actually less than $m_c/m_s = 1.35/0.175 = 7.7$ used in deducing (4). For example, if we had assumed a larger possible m_s value of 200 MeV and thus an up- and down-quark mass ratio R of 6.7, we would have $\Delta\rho(m_o = 370, m_h = 55) = 0.037$, falling within the allowed $(\rho - 1)$ region. Furthermore, there is a real possibility that the error estimate in the low-energy neutral-current data has been too optimistic[8] and when M_W and M_Z are measured precisely they might show a deviation from $\rho = 1$ larger than the limit of 0.04 as presented in Eq. (6). Thus, we shall continue the discussion of a possible fourth-generation quark in the mass ranges (3) and (4). In the following, to be definite, we shall take as representative values of fourth-generation quarks $m_o = 450$ GeV and $m_h = 60$ GeV corresponding to $R = 7.5$ and $\Delta\rho = 0.056$. The pattern of the quark mass hierarchy $m_t = 40$ GeV, $m_c = 1.35$ GeV, $m_u = 5.1$ MeV and $m_b = 5.3$ GeV, $m_s = 175$ MeV and $m_d = 8.9$ MeV will be maintained with

$$\begin{aligned} m_h : m_b : m_s : m_d &= 1 : \lambda^2 : \lambda^4 : \lambda^6 \ , \\ m_o : m_t : m_c : m_u &= 1 : \lambda^2 : \lambda^4 : \lambda^8 \ . \end{aligned} \tag{9}$$

Similarly we also have the pattern of regularity (1) which is obeyed by all generations except the first one. When combined with (9), this statement can be expressed by three relations, each representing the degree of deviation from strict proportionality between neighboring generations:

$$(m_d/m_s)^{1/2} - (m_u/m_c)^{1/2} = O(\lambda) \ , \tag{10}$$

$$(m_s/m_b)^{1/2} - (m_c/m_t)^{1/2} = A\lambda^2 \ , \tag{11}$$

$$(m_b/m_h)^{1/2} - (m_t/m_o)^{1/2} = B\lambda^2 \ , \tag{12}$$

where A and B are of order 1.

III. MIXING ANGLES FOR THE FOURTH-GENERATION QUARKS

Given the success of the Fritzsch ansatz[9] to account for the observed KM mixings in the case of three generations, we shall assume that the 4×4 quark mass matrices continue to have this structure. For $a = u,d$,

$$M^a = P^a F^a Q^a \ , \tag{13}$$

with

$$F^a = \begin{bmatrix} 0 & A^a & 0 & 0 \\ A^a & 0 & B^a & 0 \\ 0 & B^a & 0 & C^a \\ 0 & 0 & C^a & D^a \end{bmatrix}$$

being a real symmetric matrix, and P^a and Q^a are diagonal phase matrices. F^a are diagonalized by orthogonal transformation O^a. All the elements of P^a, hence also O^a, can be expressed in terms of the mass eigenvalues m_i. For the hierarchical structure displayed by the masses in Eq. (9), the orthogonal transformation can be adequately approximated by

$$O = \begin{bmatrix} 1-(m_1/2m_2) & (m_1/m_2)^{1/2} & -(m_1/m_3)^{1/2} & (m_1/m_4)^{1/2} \\ -(m_1/m_2)^{1/2} & 1-(m_1/2m_2)-(m_2/2m_3) & -(m_2/m_3)^{1/2} & (m_2/m_4)^{1/2} \\ (m_1/m_3)^{1/2}(m_2/m_3) & (m_2/m_3)^{1/2} & 1-(m_2/2m_3)-(m_3/2m_4) & (m_3/m_4)^{1/2} \\ O(\lambda^5) & O(\lambda^4) & -(m_3/m_4)^{1/2} & 1-(m_3/2m_4) \end{bmatrix} .$$

The precise expression of O_{41} and O_{42} will not be needed in the subsequent calculations as their contributions can be neglected.

Thus, the KM matrix can also be expressed in terms of m_i once the phases are given:

$$U = X(O^d)P(O^u)^T Y , \tag{14}$$

where $P = P^d P^{u*}$ and X, Y are also diagonal phase matrices expressing the rephasing freedom of the KM matrix. In the discussion of the three-generation case, we have worked out a particular model of "maximal CP violation": the CP phase is associated with the u-quark sector where the quark mass matrix relation of Eq. (1) is violated. Its natural extension leads to the phase matrix

$$P = \mathrm{diag}(-i,1,1,1) .$$

Thus, one possible realization of such a phase structure by the (normalized) mass matrices will be

$$\overline{M}_d = \begin{bmatrix} 0 & \lambda^5 & 0 & 0 \\ \lambda^5 & 0 & \lambda^3 & 0 \\ 0 & \lambda^3 & 0 & \lambda \\ 0 & 0 & \lambda & 1 \end{bmatrix} ,$$
$$\overline{M}_u = \begin{bmatrix} 0 & i\lambda^6 & 0 & 0 \\ -i\lambda^6 & 0 & \lambda^3 & 0 \\ 0 & \lambda^3 & 0 & \lambda \\ 0 & 0 & \lambda & 1 \end{bmatrix} . \tag{15}$$

In the convention we have chosen with $X = \mathrm{diag}(i,1,1,1)$ and $Y = \mathrm{diag}(-1,1,1,1)$ one finds that, to leading order in λ, the original three-generation KM matrix is not affected by the presence of the fourth. For the extra elements besides $U_{oh} = O(1)$, we have

$$\begin{aligned} U_{uh} &= i(m_u/m_t)^{1/2} B\lambda^2 - (m_d/m_h)^{1/2}(m_b/m_h) = O(\lambda^5) , \\ U_{ch} &= -(m_c/m_t)^{1/2} B\lambda^2 = O(\lambda^3) , \\ U_{th} &= B\lambda^2 = O(\lambda^2) , \\ U_{od} &= -(m_d/m_b)^{1/2} B\lambda^2 = O(\lambda^4) , \\ U_{os} &= (m_s/m_b)^{1/2} B\lambda^2 = O(\lambda^3) , \\ U_{ob} &= -B\lambda^2 = O(\lambda^2) , \end{aligned} \tag{16}$$

where the $O(1)$ parameter B is defined in Eq. (12). The order of magnitude of the KM elements (whether real or imaginary) is summarized below:

$$U = \begin{bmatrix} 1 & \lambda & \lambda^4 & \lambda^5 \\ \lambda & 1 & \lambda^2 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^3 & \lambda^2 & 1 \end{bmatrix} . \tag{17}$$

Again we note that the mixings between neighboring generations typically are of the order λ^2. The only exception is the anomalously large Cabibbo angle, reflecting the anomalously light u quark so there is not much cancellation in Eq. (10). Furthermore, since each term in the left-hand side (LHS) of Eqs. (11) is larger than that of (12), as the $1/\lambda^2$ increase to the fourth generation must be moderate in order to avoid a strong violation of the ρ-parameter constraint, we would expect that this will be reflected in a slight increase in the value of B as compared to A. Thus we would expect the mixing between the fourth and the third generations to be somewhat larger than that between the third and the second. This expectation differs markedly from the often stated speculation of progressive suppression of mixings between neighboring generations as quarks get heavier.

IV. PHENOMENOLOGICAL IMPLICATIONS: PRODUCTION, DECAY, AND THE LIFETIME OF HEPTA-QUARK

The prospects of production and detection for the charged κ lepton and the superheavy o quark represent two antipodes: For κ with a mass in the 20—30-GeV range this presumably will be fairly similar to the case of the τ lepton and the question of whether such a particle exists or not can be decided upon by the upcoming e^+e^- collider TRISTAN at KEK (Ref. 10). On the other hand, the octa-quark with a mass in the high-400-GeV range is beyond the reach of any accelerator facilities presently under construction. In this section we shall, therefore, discuss mainly the case of the hepta-quark with a mass around 60 GeV (Ref. 11).

For heavy quarks beyond charm and beauty and the constituent weak decay rate ($\propto m_q{}^5$) so greatly overwhelms the lepton decays of the heavy quarkonium ($\propto m_q$) that we cannot rely on detecting the latter in a Drell-Yan process. Thus the existence of the top quark is signaled by the chain

$$\begin{aligned} &p\bar{p} \rightarrow WX \\ &\quad \hookrightarrow \; t\bar{b} \\ &\qquad \hookrightarrow \; be\bar{\nu} . \end{aligned}$$

However, such a scenario will not be applicable for the h quark: the mixing-angles favored modes of $W \rightarrow h\bar{o}$ and $h\bar{t}$ are kinematically not allowed and the first allowed channel of $W \rightarrow h\bar{c}$ is already strongly suppressed with $U_{ch} = O(\lambda^3)$. Instead of W decays we must therefore rely on the inclusive $h\bar{h}$ pair production. The key will then be finding signals rising above the QCD background. In this connection clearly the more relevant modes are the semileptonic decays. Let us concentrate first on the decays into electron channels: Since the h quark is heavy compared with the typical strong-interaction scale, the rates can be approximated by free quark rates. Thus

$$\begin{aligned} \Gamma(h \rightarrow e\bar{\nu}X) \simeq & \, \Gamma(h \rightarrow e\bar{\nu}t) \\ & + \Gamma(h \rightarrow e\bar{\nu}c) \\ & + \Gamma(h \rightarrow e\bar{\nu}u) \end{aligned} \tag{18}$$

with

$$\begin{aligned} \Gamma(h \rightarrow e\bar{\nu}t) &\simeq |U_{th}|^2 R_h f(m_t/m_h) , \\ \Gamma(h \rightarrow e\bar{\nu}c) &\simeq |U_{ch}|^2 R_h , \\ \Gamma(h \rightarrow e\bar{\nu}u) &\simeq |U_{uh}|^2 R_h , \end{aligned}$$

where

$$R_h = \frac{G_F{}^2 m_h{}^5}{192\pi^3} ,$$

and

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x .$$

Since $U_{th} = O(\lambda^2)$, $U_{ch} = O(\lambda^3)$, $U_{uh} = O(\lambda^5)$, and $f(m_t/m_h) = O(\lambda^2)$ for $m_t \simeq 40$ GeV and $m_h \simeq 60$ GeV (Ref. 12), the u channel will be totally negligible while top and charm branching ratios are comparable, $\Gamma_{c,t} \approx \lambda^4 \times 4 \times 10^{19}$ sec$^{-1} \approx 10^{17}$ sec^{-1}. Thus even with a conservative estimate of the branching ratio into electron channels to be $\simeq 10\%$, the lifetime of the lepton meson is at most of 10^{-18} sec and its decay lengths will be much too short to be measurable by any vertex detector. Thus our conclusion is quite different from the suggestions made by a number of authors in this area. It of course reflects the different expectations one has as to how the KM matrix will be extended to the fourth generation.

V. PHENOMENOLOGICAL IMPLICATIONS: OCTA-QUARK CONTRIBUTION IN HIGHER-ORDER PROCESSES

None of the accelerator facilities in existence, or under construction, has high enough energy to produce the octa-quark which our model suggests is likely to have a mass greater than 400 GeV. However, its presence may be indicated through effects on the low-energy phenomenology via the higher-order virtual processes. Here we shall discuss the possible contribution by the o quark to the CP-violation parameters ϵ and ϵ' and the flavor-changing decay mode of the Z intermediate boson.

The kaon CP impurity parameter ϵ is dominated by the short-distance physics as represented by the box diagram involving the exchange of W bosons and heavy quarks. To compare the strength of contribution by the o quark to those of the lower generation charm and top flavors, we write

$$\epsilon \propto \sum_{i,j=c,t,o} \eta_{ij} \mathrm{Im}\lambda_i \lambda_j E(x_i, x_j) , \tag{19}$$

where η_{ij} are the QCD correction coefficients, λ_i is the mixing-angle product

$$\lambda_i = U_{si} U_{di}^* , \tag{20}$$

and $E(x_i, x_j)$ with $x_i = m_i{}^2/M_W{}^2$ is the kinematic factor[13]

$$E(x_i,x_j) = x_i x_j \left[\left[\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2} \right] \frac{\ln x_i}{x_i - x_j} + (x_i \leftrightarrow x_j) - \frac{3}{4(1-x_i)(1-x_j)} \right] \tag{21a}$$

which for $i = j$ turns into

$$E(x_i,x_i) = x_i \left[\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} \right] - \frac{3}{2} \left[\frac{x_i}{1-x_i} \right]^3 \ln x_i . \tag{21b}$$

The QCD factors have a weak dependence on flavors: $\eta_{cc} \approx 0.7$, $\eta_{tt} \approx 0.6$, and $\eta_{ct} \approx 0.4$ and η_{oo}, η_{ot}, and η_{co} are not expected to be significantly smaller.[14] So we shall concentrate on the two competing factors of the mixing-angle product $\mathrm{Im}\, U_{si} U_{di}^* U_{sj} U_{dj}^*$ which in this Fritzsch model decreases rapidly for heavier flavors i and j, and the kinematic factor $E(x_i, x_j)$ in Eq. (21) which increases strongly with quark masses. For the lower three generation with $x_c \ll 1$ and $x_t < 1$ we have

$$x_c{}^{-1} E(x_c, x_c) \simeq 1 , \tag{22}$$

$$x_c{}^{-1} E(x_t, x_t) \simeq x_t / x_c = O(\lambda^{-4}) , \tag{23}$$

and

$$x_c{}^{-1} E(x_c, x_t) \simeq \ln(x_t/x_c) = O(\lambda^{-1}) . \tag{24}$$

For the fourth generation with $x_0 \simeq 30$ corresponding to a $m_o \simeq 450$ GeV,

$$x_c{}^{-1} E(x_o, x_o) \simeq x_o / (4x_c) = O(\lambda^{-7}) , \tag{25}$$

$$x_c{}^{-1} E(x_o, x_c) \simeq -\ln x_c = O(\lambda^{-1}) , \tag{26}$$

and

$$x_c{}^{-1} E(x_o, x_t) \simeq (x_t/x_c)[-\ln x_t + (\ln x_o)/4 + \tfrac{3}{4}] = O(\lambda^{-5}) . \tag{27}$$

For the mixing-angle products, besides the previous three-generation result of

$$\mathrm{Im}\lambda_c{}^2 = -2\, \mathrm{Im}\lambda_c \lambda_t = 2A^2 \eta \lambda^6 = O(\lambda^7) , \tag{28}$$

$$\mathrm{Im}\lambda_t{}^2 = -2A^4 \eta \lambda^{10} = O(\lambda^{11}) , \tag{29}$$

we also need to calculate $\mathrm{Im}\lambda_o{}^2$, $\mathrm{Im}\lambda_o \lambda_c$, and $\mathrm{Im}\lambda_o \lambda_t$. For this we choose the convention of keeping, among the new fourth-generation KM elements, U_{os} and U_{oh} real. The imaginary part of U_{od} can then be fixed by unitarity:

$$U_{os} = Ba\lambda^3 ,$$

$$U_{od} = (-a + i\xi) B \lambda^4 ,$$

where besides the parameter B of Eq. (12) we have introduced new notations of

$$a\lambda = (m_s/m_b)^{1/2} ,$$

$$\xi \lambda^2 = (m_u/m_t)^{1/2} .$$

Thus B and a are $O(1)$ parameters and ξ is $O(\lambda)$. From this we obtain

$$\lambda_0 = U_{os}{}^* U_{od} = (-a + i\xi) B^2 a \lambda^7 .$$

This together with our previous result[2] of

$$\lambda_c = -\lambda - i a^2 \eta \lambda^5 ,$$

$$\lambda_t = -(1 - \sigma + i\eta) A^2 \lambda^5 ,$$

and the knowledge that A, B, and a are $O(1)$ parameters while σ, η, and ξ are $O(\lambda)$, leads to the new mixing-angle factors:

$$\text{Im}\lambda_o{}^2 = 2B^4a^3\xi\lambda^{14} = O(\lambda^{15}) , \quad (30)$$

$$\text{Im}\lambda_o\lambda_c = -B^2a\xi\lambda^8 = O(\lambda^9) , \quad (31)$$

$$\text{Im}\lambda_o\lambda_t = -A^2B^2a(a\eta+\xi)\lambda^{12} = O(\lambda^{13}) . \quad (32)$$

Combining Eqs. (28)–(32) with Eqs. (22)–(27) we can estimate the contribution from each of the exchanged heavy-quark pairs. Normalizing the terms in the ϵ sum by $m_c{}^2$ we have, for the product,

$$\tilde{\epsilon}(i,j) = x_c{}^{-1}E(i,j)\text{Im}\lambda_i\lambda_j ,$$

from the first three generations $\tilde{\epsilon}(c,t)=O(\lambda^6)$ and $\tilde{\epsilon}(c,c),\tilde{\epsilon}(t,t)=O(\lambda^7)$, and from the fourth generation $\tilde{\epsilon}(o,o),\tilde{\epsilon}(o,c),\tilde{\epsilon}(o,t)=O(\lambda^8)$. Thus the additional fourth-generation contributions are not expected to dominate.[15] However, since an expansion parameter λ is not very small, these additional terms are not altogether insignificant, especially when the uncertainty in the calculation of the matrix element of the $\Delta S=2$ operator between the K^0 and $\bar{K}^0$ states (the "B parameter") can be reduced below the 20% level.

We next comment briefly on the ϵ'-parameter calculation. If the usual assumption of penguin diagram dominance is indeed correct then the fourth-generation contribution will be negligible. Here the mixing-angle product $U_{is}^*U_{id}$ is multiplied by a kinematic factor that does not increase significantly with the mass of the intermediate heavy quark. Thus the mixing-angle suppression completely determines the final result.

Another comment we wish to make concerns the possibility of heavy-quark-induced flavor-changing Z decay. Ganapathi *et al.*[16] have shown that the amplitude for $Z\to\bar{b}h$ through the triangle diagram with W exchange between an intermediate superheavy octa-quark pair will be (much like the case of the box diagram considered above), proportional to

$$U_{oh}U_{ob}^*m_o{}^2/M_W{}^2 .$$

For KM mixing $|U_{oh}U_{ob}^*|=O(\lambda^2)$ and $m_o\simeq 450$ GeV the branching ratio can be as high as 10^{-5} which will be on the boarder of observability at the "Z factory" of the Stanford Linear Collider and CERN LEP.

The conclusion is that if the fourth generation exists, its charge $\frac{2}{3}$ quark must be superheavy, in the 400–500-GeV range. Thus even with small mixings as suggested by the Fritzsch ansatz its effects on the low-energy phenomenology through the higher-order loop diagrams, although not dominant, may still be fairly significant. In this connection the renormalization-group analysis of Bagger, Dimopoulos, and Massó[17] indicates that such new quarks would imply new physics below 10 TeV.

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[10]To be sure, a 20–30-GeV charged lepton can already be produced in existing $\bar{p}p$ collider facilities via the W-decay chain. However, here the κ signal does not exceed the background and a clean and decisive identification of the κ lepton will most likely be achieved in the upcoming high-energy e^+e^- collider. For a discussion of production and detection of a charged lepton in $\bar{p}p$ collisions see Barger, Baer, Hagiwara, and Phillips in Ref. 1.

[11]See for example, Barger, Baer, Hagiwara, and Phillips in Ref. 1.

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[15]Recall that the QCD suppression factors η_{ij} do not differ from each other significantly. However, because λ is not very small, if the λ-suppressed term receives a smaller QCD correction η_{ij} may blur the distinction between terms differing, say by one power of λ, e.g., numerically the (c,t) contribution is actually comparable to those of (c,c) and (t,t). In the case of the fourth generation the λ- and η-suppression effects go in the same direction—QCD renormalization also tends to suppress the heavy-particle contributions.

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