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Mass-matrix ansatz and flavor nonconservation in models with multiple Higgs doublets

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Motivated by the existence of three fermion generations, we consider models with three Higgs doublets (for each hypercharge). It is shown that, if the basic Yukawa coupling matrices are of the Fritzsch form or some generalizations thereof (two such examples are presented), then the neutral flavor-changing couplings have a hierarchical structure given by $\Delta_{ij}\sqrt{m_i m_j}$, where Δ_{ij} is a combination of mixing angles and the m_i's are the relevant fermion masses. These couplings give a lower bound of the order of Δ_{ds} times 1 TeV on the mass of the exchanged scalar from the neutral-kaon mass difference. If the Higgs-boson mass is not significantly greater than this limit, one expects $D\bar{D}$ mixing comparable to the present experimental bound.

I. INTRODUCTION

Phenomenologically, the standard model of an SU(3) $\times$SU(2)$\times$U(1) gauge theory with three generations of leptons and quarks has been very successful. In the minimal standard model, it is assumed that there is a single doublet of Higgs scalars. Its vacuum expectation value (VEV) breaks the gauge symmetry giving masses to the gauge bosons and fermions. Although it still leaves all fermion masses and mixing angles as free parameters of the theory, the isodoublet character of the Higgs particle leads to the correct intermediate-vector-boson mass relation $M_W = M_Z\cos\theta_W$. In this paper we shall consider models with multiple doublets of Higgs particles.

Models with two Higgs doublets of opposite hypercharge, coupling to the $I_3 = +\frac{1}{2}$ and $I_3 = -\frac{1}{2}$ fermions, respectively, have often been discussed. Both doublets are needed in all supersymmetric models (to give mass to all the quarks) and both are needed in all models in which a global U(1) (Peccei-Quinn) symmetry is used to solve the strong *CP* problem.[1] In this paper, however, we will be interested in models in which several Higgs doublets of the *same* hypercharge exist. These doublets can, in general, all couple to the same quarks and leptons. Of course, one could still have a supersymmetric model or one with a Peccei-Quinn symmetry by having two sets of several doublets, each set with opposite hypercharges.

Why should one suppose that several "generations" of Higgs doublets exist? The first reason is primarily aesthetic. No known principle restricts the number of generations of fermions, and at least three such generations are known to exist. Similarly, no known principle restricts the number of generations of Higgs doublets, thus several generations could certainly exist. For simplicity, we will assume that the number of such generations (for a given hypercharge) is identical to the number of fermion generations.

The second motivation comes from the current enthusiasm for the superstring theory. When the ten-dimensional $E_8\times E_8'$ heterotic superstring theory[2] is compactified down to four dimensions, the resulting low-energy theory[3] is a subgroup of E_6 with the observed quarks and leptons in each generation identified with 15 members of the **27** representation of E_6, which decomposes with respect to SO(10) and SU(5) as

$$\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}$$
$$= (\mathbf{5} + \mathbf{10}^* + \mathbf{1}) + (\mathbf{5} + \mathbf{5}^*) + \mathbf{1} \ .$$

Thus, for each generation, besides the right-handed neutrino, we have the additional fields in a $\mathbf{5} + \mathbf{5}^* + \mathbf{1}$ of SU(5). The **5** (**5***) contains an $SU(2)_L$ doublet with hypercharge $+1$ (-1). The supersymmetric scalar partners of these particles have the quantum numbers of the standard Higgs doublets of the supersymmetric standard model. Clearly, in such models the number of generations of Higgs doublets is the same as the number of generations of quarks and leptons.[4] In phenomenological analyses of the low-energy limit of superstring theory, it is generally assumed that only one generation of Higgs scalars couples to quarks and leptons (this assumption is often masked by calling the other generations by another name[5]). Of course, the hope is that such an assumption will turn out to be justified for geometric or topological reasons. Nonetheless, several generations of Higgs scalars do exist in E_6-type models, and in general they will all couple to fermions.

It is often stated that multiple Higgs scalars, especially in the sense of multigenerational Higgs scalars, will lead to the problem of predicting too large a flavor-changing neutral-current effect. We examine the assumptions underlying this statement in Sec. II. This will motivate our consideration of a set of phenomenological Yukawa couplings which naturally suppress the flavor-changing effects among light fermions. In Sec. III we consider a simple example of such a coupling scheme. It leads to fermion mass matrices of the Fritzsch type[6] [which have had some phenomenological success in their prediction of

Kobayashi-Maskawa (KM) mixing angles in terms of the quark masses[7]]. We emphasize in particular that such a Yukawa coupling scheme will lead to a $D\bar{D}$ mixing at a level that is orders of magnitude larger than the standard-model prediction, at a level in fact comparable to the present experimental upper limit. Other schemes of Yukawa couplings that lead to fermion mass matrices that are generalizations of the Fritzsch type will be examined in Sec. IV, and in Sec. V we present our conclusions.

II. FLAVOR-CHANGING NEUTRAL CURRENTS

We first turn to the question of flavor-changing neutral currents in multiple-generation models. For simplicity, we shall assume that there is only one doublet (coupling to all quarks) per generation. If there are two doublets per generation (as in supersymmetric models), then our Yukawa couplings should be multipled by a ratio of vacuum values and by a mixing angle between the weak and mass eigenstates of the two doublets. Since these factors are arbitrary (but not too different from 1), we will ignore them; but one should keep in mind that, in models with two Higgs doublets per generation, the couplings have an unknown factor of ~ 1 multiplying them.

The basic Yukawa couplings give rise to fermion mass terms as

$$H_i^0 \bar{q}'_{jL} \lambda'_{ijk} q'_{kR} \xrightarrow[\langle H_i^0 \rangle = v_i/\sqrt{2}]{} \bar{q}'_{jL} (v_i \lambda'_{ijk}/\sqrt{2}) q'_{kR} , \tag{1}$$

where q's are the quark fields of the same charge. We have not bothered to display all the details of the real/imaginary (hence, scalar/pseudoscalar) components of the Higgs fields H_i^0. The vacuum expectation values are fixed by the weak scale

$$\sum_i v_i^2 \equiv v^2 = (\sqrt{2} G_F)^{-1} \simeq (250 \text{ GeV})^2 . \tag{2}$$

We diagonalize the quark mass matrix $M'_{jk} \equiv \sum_i v_i \lambda'_{ijk}/\sqrt{2}$ with respect to quark weak eigenstates $q'_{L,R}$,

$$q'_L M' q'_R = q_L M q_R , \tag{3}$$

by transforming the fields to mass eigenstates $q_{L,R}$

$$q'_{L,R} = O q_{L,R} \tag{4}$$

with[8]

$$O^T M' O = \sum_{j,k} (O^T)_{lj} \left[\sum_i v_i \lambda'_{ijk}/\sqrt{2} \right] (O)_{kn} = M_{ln} = m_l \delta_{ln} . \tag{5}$$

The Yukawa couplings of immediate physical interest are those defined with respect to the quark mass eigenstates

$$\lambda_{iln} = \sum_{j,k} (O^T)_{lj} \lambda'_{ijk} (O)_{kn} . \tag{6}$$

It is useful to scale these couplings by a mass factor so we can associate with the Yukawa couplings of the ith Higgs particle the "ith mass matrix" $M^{(i)}$ defined as

$$M_{\text{ln}}^{(i)} = v_i \lambda_{iln}/\sqrt{2} \quad \text{index } i \text{ not summed} . \tag{7}$$

When we sum over the Higgs generation index i, the result should reproduce the diagonalized quark mass matrix

$$\sum_i M_{ln}^{(i)} = M_{ln} = m_l \delta_{ln} . \tag{8}$$

If the VEV's (for all i) are comparable, $v_i \approx v$ of Eq. (2), we expect that the Yukawa coupling matrix to have the order of magnitude

$$\lambda_{iln} \approx \frac{\sqrt{2}}{v} M_{ln}^{(i)} = 2^{3/4} G_F{}^{1/2} M_{ln}^{(i)} . \tag{9}$$

In the minimal standard model with one Higgs doublet, clearly the single coupling matrix must equal the final fermion mass matrix and must therefore be flavor conserving.

Any model with several Higgs particles faces the problem of unsuppressed flavor-changing couplings and unacceptably large rates for processes such as $\mu \to e\gamma$, $\mu \to 3e$, $K \to \mu\mu$, $K \to \mu e$, and too large a value for $\Delta m_K = m_{K_L} - m_{K_S}$, etc. Since it is generally recognized that the most stringent limit comes from Δm_K (Ref. 9), we shall mainly discuss this quantity:

$$\Delta m_K = 2 \langle \bar{K} \,|\, \mathscr{L}^{\Delta S=2} \,|\, K \rangle . \tag{10}$$

For the effective $\Delta S = 2$ coupling, the flavor-changing Higgs-boson-exchange tree diagram yields an amplitude[10]

$$\mathscr{L}_{\text{Higgs}}^{\Delta S=2} = \sum_i \sqrt{2} G_F (M_{sd}^{(i)})^2 (m_H)_i{}^{-2} (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) , \tag{11}$$

where $M_{sd}^{(i)}$ is the dimensional Yukawa coupling, defined in Eq. (7), between s,d quarks and the ith Higgs particle with mass $(m_H)_i$. We have dropped the scalar coupling $(\bar{s}d)(\bar{s}d)$ in Eq. (11) because it has negligibly small matrix elements when compared to the pseudoscalar term, which according to a bag-model calculation[11] yields

$$\langle \bar{K} \,|\, (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \,|\, K \rangle \simeq 8.5 \times 10^{-2} \text{ GeV}^3 . \tag{12}$$

Lacking any specific knowledge to distinguish one Yukawa coupling matrix $M^{(i)}$ from others, one usually makes the assumption that they are all similar in structure $M^{(i)} \simeq \bar{M}$ for all i and define the characteristic Higgs-boson mass m_H:

$$\frac{\bar{M}_{jk}{}^2}{m_H{}^2} \equiv \sum_i \frac{M_{jk}^{(i)2}}{(m_H{}^2)_i} . \tag{13}$$

As for the dimensional Yukawa coupling matrix $\bar{M}$ itself, some authors advocate the following estimate: since the most conservative approach is to take all elements $\bar{M}_{jk}$ being comparable, and since in some sense the heaviest fermion sets the scale for the whole matrix, we can assume each element $\bar{M}_{jk}$ being a product of this mass scale and some mixing angle factor. As we do not know these mixing angle factors we shall set all of them to 1. Thus $\bar{M}_{sd}$ is set to the b-quark mass m_b:[12]

$$\sum_i \frac{M_{sd}^{(i)2}}{(m_H{}^2)_i} \simeq \frac{m_b{}^2}{m_H{}^2} . \tag{14}$$

In this way, one derives the lower bound for the flavor-changing Higgs-boson mass[13]

$$m_H{}^2 > \frac{2\sqrt{2}G_F m_b{}^2}{\Delta m_K} \times 8.5\times 10^{-2}\ \text{GeV}^3 \simeq (150\ \text{TeV})^2 . \tag{15}$$

Is this a reasonable estimate of the lower bound for the characteristic Higgs-boson mass? We suggest that it is not. The key assumption in the above derivation is that all elements of the Yukawa mass matrix $\overline{M}_{jk}$ are of comparable magnitude, and are of the order of the heaviest fermion mass (times a mixing-angle factor of order unity), thus the approximation $\overline{M}_{sd} \simeq m_b$. This does not seem to us to be a reliable estimate. After all, one of the most conspicuous features of the fermion mass spectrum is its hierarchical structure, e.g., $m_d \ll m_s \ll m_b$, etc. (In the standard model with a single Higgs doublet the mass matrix $M_{jk} = m_j \delta_{ik}$ of course exhibits this hierarchical structure on its diagonal.) We are interested in nondiagonal $\overline{M}_{jk}$ with flavor nonconservation. We shall demonstrate that in a broad class of models with phenomenologically sound fermion mass matrices M'_{jk}, the Yukawa coupling matrices have the general hierarchical structure of

$$\overline{M}_{jk} = \Delta_{jk}\sqrt{m_j m_k} , \tag{16}$$

where Δ_{jk} denotes various mixing-angle factors (some of which were discussed earlier). Even taking Δ_{sd} to be unity the relevant coupling $\overline{M}_{sd}$ for Δm_K calculation is reduced by a factor of $(m_s m_d)^{1/2}/m_b \simeq 7\times 10^{-3}$. Thus the Δm_K lower bound for the Higgs-boson mass correspondingly decreases by this factor, down to the much more reasonable neighborhood of 1 TeV. Should Δ_{sd} be somewhat less than unity, the bound would be correspondingly lower.

Since the Higgs-boson self-coupling in the minimal standard model is proportional to $G_F m_H{}^2$ it has been frequently remarked that $m_H \gtrsim 1$ TeV implies strong interaction among Higgs bosons. Thus if considerations are restricted to the standard model, one notes that tree-diagram partial-wave unitarity breaks down[14] and weak interactions become strong. Of course, given the uncertainties in Δ_{sd}, the lower bound could be significantly lower than 1 TeV; thus, strongly interacting Higgs bosons might not be a problem. Even if the bound is 1 TeV, however, if the standard model is only the low-energy effective theory of a larger edifice, this could be viewed as the threshold effect of some new physics. For example, if the Higgs scalars are bound states, then all of the above results would still apply, but the natural scale for the Higgs-boson masses would be 1 TeV, and the bound on Δm_K would not be a problem. Alternatively, given our motivation for considering multigenerational Higgs scalars as the supersymmetric scalars in the **27** of E_6 models, this new physics could be the opening up of supersymmetric particle thresholds. In this case, the natural scale for all of the Higgs-boson masses but one[15] is also ~ 1 TeV (the one relatively light Higgs boson could have a relatively small value of Δ_{sd}). Thus, ~ 1 TeV Higgs-boson masses are not unreasonable.

If Higgs-boson masses are not significantly greater than 1 TeV, even though their contribution to $K\overline{K}$ mixing may not be important, it is likely to bring about a $D\overline{D}$ mixing that is much larger than the standard-model prediction.[16] Let us recall that the usual Glashow-Iliopoulus-Maiani—(GIM) suppressed box diagrams for the effective $\Delta S = 2$ and $\Delta C = 2$ interactions involve very different quark masses, resulting in a Δm_D much smaller than Δm_K:

$$\left[\frac{\Delta m_D}{\Delta m_K}\right]_{\text{SM}} \simeq \left[\frac{m_s{}^2 - m_d{}^2}{m_c{}^2 - m_u{}^2}\right] \frac{\langle \overline{D} \mid (\bar{c}\gamma_\lambda\gamma_5 u)(\bar{c}\gamma^\lambda\gamma_5 u) \mid D\rangle}{\langle \overline{K} \mid (\bar{s}\gamma_\lambda\gamma_5 d)(\bar{s}\gamma^\lambda\gamma_5 d) \mid K\rangle} , \tag{17}$$

where we have ignored the contribution due to the third-generation fermions because of their small mixings. The ratio of the matrix elements can be reliably estimated to be $(f_D/f_K)^2(m_D/m_K) \simeq 15$ by taking the D-meson decay constant $f_D \simeq 300$ MeV (Ref. 17). Thus the standard model yields

$$\left[\frac{\Delta m_D}{\Delta m_K}\right]_{\text{SM}} \simeq 0.2 . \tag{18}$$

Namely, $\Delta m_D \simeq 7\times 10^{-16}$ GeV.

For Higgs-boson exchange with the flavor-changing coupling $\overline{M}_{jk}$ proportional to $\sqrt{m_j m_k}$, we have a very different quark mass dependence:

$$\left[\frac{\Delta m_D}{\Delta m_K}\right]_{\text{Higgs}} \simeq \left[\frac{m_c m_u}{m_s m_d}\right] \frac{\langle \overline{D} \mid (\bar{c}\gamma_5 u)(\bar{c}\gamma_5 u) \mid D\rangle}{\langle \overline{K} \mid (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \mid K\rangle} . \tag{19}$$

Again if we estimate the matrix elements by the vacuum-insertion method—recall its good agreement with the bag-model calculation[11] (we expect it should be even more reliable for the ratio of two matrix elements),

$$\frac{\langle \overline{D} \mid (\bar{c}\gamma_5 u)(\bar{c}\gamma_5 u) \mid D\rangle}{\langle \overline{K} \mid (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \mid K\rangle} \simeq \frac{f_D{}^2}{f_K{}^2}\left[\frac{m_D}{m_K}\right]^3 \left[\frac{m_s + m_d}{m_c + m_u}\right]^2 \simeq 4 . \tag{20}$$

Even though this ratio does not quite favor the Δm_D as in the axial-vector case we still get

$$\left[\frac{\Delta m_D}{\Delta m_K}\right]_{\text{Higgs}} \simeq 20 . \tag{21}$$

Thus if the Higgs-boson-exchange mechanism saturates the Δm_K bound we would have $(\Delta m_D)_{\text{Higgs}} \simeq 7\times 10^{-14}$ GeV, or, more generally,[18]

$$(\Delta m_D)_{\text{Higgs}} \simeq 7\times(\text{TeV}/m_H)^2 \times 10^{-14}\ \text{GeV} . \tag{22}$$

This should be compared to the present experimental limit of $\Delta m_D < 6.5\times 10^{-13}$ GeV (Ref. 19). In any case the point we are making is that the Higgs-boson-exchange mechanism can potentially account for a much larger $D\overline{D}$ mixing than the standard model. (Note that $T\overline{T}$ mixing would be enormous, with $\Delta m_T \approx 10^{-10}$ GeV.)

III. AN ANSATZ FOR YUKAWA COUPLING MATRICES: THE SIMPLE FRITZSCH SCHEME

An example of the (weak-eigenstate) fermion mass matrix M' that summarizes well the phenomenology of fermion masses and mixings is the Fritzsch mass matrix

$$M' = \begin{bmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{bmatrix} . \tag{23}$$

Expressing the matrix elements in terms of the mass eigenvalues, we have, in the limit of $m_3 \gg m_2 \gg m_1$,

$$A \simeq (m_1 m_2)^{1/2}, \quad B \simeq (m_2 m_3)^{1/2}, \quad C \simeq m_3 . \tag{24}$$

The Hermitian matrix M' is diagonalized by the orthogonal transformation $(O^T M' O)_{ij} = m_i \delta_{ij}$ with

$$O \simeq \begin{bmatrix} 1 & (m_1/m_2)^{1/2} & -(m_1/m_3)^{1/2} \\ -(m_1/m_2)^{1/2} & 1 & -(m_2/m_3)^{1/2} \\ 0 & (m_2/m_3)^{1/2} & 1 \end{bmatrix} . \tag{25}$$

When both the charge $-\frac{1}{3}$ quark and charge $+\frac{2}{3}$ quark mass matrices have the Fritzsch form, the resulting Kobayashi-Maskawa (KM) mixing $V = O_u O_d^T$ will also be close to the identity matrix with small nondiagonal elements:

$$V_{ud} \simeq V_{cs} \simeq (m_d/m_s)^{1/2} - (m_u/m_c)^{1/2} \simeq (m_d/m_s)^{1/2} \tag{26}$$

and

$$V_{cb} \simeq (m_s/m_b)^{1/2} - (m_c/m_t)^{1/2} . \tag{27}$$

Thus the Cabibbo angle $\theta_C \simeq V_{ud} \simeq V_{cs} \simeq 0.225$ comes out correctly, and the experimental value of $V_{cu} \simeq 0.06$ can also be accounted for with an m_t in the reasonable range of 20 to 80 GeV. Namely, for the Fritzsch mass matrix ansatz to account for the observed hierarchy among the KM mixing angles there must be an appropriate cancellation between the two large terms in Eq. (27). [This cancellation is not too delicate; the first term in Eq. (27) being only $\frac{5}{2}$ times V_{cb}.] Fortunately this "fine-tuning" can be interpreted simply as reflecting an approximate constancy of the quark mass ratio in each generation: $m_s/m_c \simeq m_b/m_t$, which may result from the approximate proportionality of the entire M_u' and M_d' mass matrices.[20] We should also note the Fritzsch mass matrix prediction for the KM element V_{ub},

$$|V_{ub}/V_{cb}| \simeq (m_u/m_c)^{1/2} \simeq 0.06 ,$$

and thus the branching ratio,

$$R \equiv \Gamma(b \to ue\nu)/\Gamma(b \to ce\nu) \simeq 0.006 , \tag{28}$$

is also compatible with the present experimental upper limit of $R < 0.04$ (Ref. 21).

In this section we shall study an ansatz for Yukawa coupling matrices (defined with respect to the fermion weak eigenstates) which gives rise to mass matrices of the Fritzsch form (23) and which, as we shall see, yields the desired hierarchical coupling matrices (defined with respect to the fermion mass eigenstates) with suppressed flavor-changing elements for the light generations. In this scheme, "the simple Fritzsch scheme," all the weak eigenstate Yukawa matrices are prescribed to have the Fritzsch form:

$$(\lambda_i')_{jk} = \frac{\sqrt{2}}{v_i} \begin{bmatrix} 0 & A_i & 0 \\ A_i & 0 & B_i \\ 0 & B_i & C_i \end{bmatrix} \tag{29}$$

with the elements having roughly the same hierarchical structure as the mass matrix itself:

$$A_i = a_i (m_1 m_2)^{1/2}, \quad B_i = b_i (m_2 m_3)^{1/2}, \quad C_i = c_i m_3 , \tag{30}$$

where the coefficients (a_i, b_i, c_i) are of order 1. Namely, the $M' = \sum_i v_i \lambda_{ijk}' / \sqrt{2}$ requirements, Eqs. (6)–(8),

$$\sum_i a_i = 1, \quad \sum_i b_i = 1, \quad \sum_i c_i = 1 , \tag{31}$$

are satisfied not through any strong cancellations among the different Yukawa couplings.[22] This is certainly the simplest ansatz one can make to reproduce the Fritzsch mass matrix. In other words the Fritzsch mass matrix M' in Eq. (23) being the sum (over the Higgs generation index i) of the Yukawa coupling matrices in Eq. (29), the simplest way to have the Fritzsch zeros $M_{11}' = M_{22}' = M_{13}' = M_{31}' = 0$ without fine-tuning is to have zeros located at the same positions in each of the Yukawa coupling matrices initially.[23]

Using the expression for the orthogonal transformation given in Eq. (25) we can immediately work out the Yukawa coupling matrices for the mass eigenstates:

$$(\lambda_i)_{jk} = \sum_{a,b} O_{ja} O_{kb} (\lambda_i')_{ab}$$

or, in terms of the dimensional coupling in Eq. (7),

$$M^{(i)} = \begin{bmatrix} (2a_i - 2b_i + c_i) m_1 & (a_i - 2b_i + c_i)\sqrt{m_1 m_2} & (b_i - c_i)\sqrt{m_1 m_3} \\ (a_i - 2b_i + c_i)\sqrt{m_1 m_2} & (-2b_i + c_i) m_2 & (b_i - c_i)\sqrt{m_2 m_3} \\ (b_i - c_i)\sqrt{m_1 m_3} & (b_i - c_i)\sqrt{m_2 m_3} & c_i m_3 \end{bmatrix} . \tag{32}$$

This result satisfies the requirement of Eq. (8) and has the hierarchical feature of

$$M_{jk}^{(i)} = \Delta_{jk} (m_j m_k)^{1/2} \tag{33}$$

and thus the lower bound on the mass of the Higgs scalars could easily be much less than 1 TeV (Ref. 24).

IV. OTHER EXAMPLES OF YUKAWA COUPLING MATRICES WITH HIERARCHICAL STRUCTURE

In the preceding section it was noted that models in which the mass matrix is of the Fritzsch form allow the successful prediction of all mixing angles in terms of masses, and it was shown that multiple Higgs models in which the Yukawa couplings were of the Fritzsch form have neutral flavor-changing couplings of the form $\Delta_{jk}\sqrt{m_j m_k}$.

Let us restate the Fritzsch ansatz: the only nonzero elements of the mass matrix, M'_{jk}, occur when (a) $j=k\pm1$ or (b) $j=k=\alpha$, where α is chosen to be 3. In models with multiple doublets, however, the ansatz must be generalized to a third-rank tensor λ'_{ijk} rather than the second-rank tensor M'_{jk}. The "simple" Fritzsch ansatz of the last section postulates that λ'_{ijk}, for each Higgs generation index i, has the identical structure as the M'_{jk} in Fritzsch forms: the only nonzero elements of λ'_{ijk} occur when (a) $j=k\pm1$ or (b) $j=k=\alpha$, where α is chosen to be 3. Here, we consider other generalizations, which we term "cyclic" Fritzsch and "extended" Fritzsch couplings.

A. The cyclic Fritzsch scheme

In the simple Fritzsch scheme, the choice of the nonzero diagonal element was the same for each Yukawa coupling, i.e., α was independent of i. It would seem more natural, if the Higgs scalars are associated with fermion generations, to have the choice of nonzero diagonal element depends on the fields to which the Higgs scalars couple. The cyclic Fritzsch ansatz says that $\lambda'_{ijk}\neq0$ when (a) $j=k\pm1$ or (b) $i=j=k$. Such an ansatz treats the Higgs scalars more equally and does not pick out one generation as special. The Yukawa couplings are thus

$$\frac{H_1}{v_1}\bar{q}'\begin{bmatrix}E_1 & A_1 & 0\\ A_1 & 0 & B_1\\ 0 & B_1 & 0\end{bmatrix}q'+\frac{H_2}{v_2}\bar{q}'\begin{bmatrix}0 & A_2 & 0\\ A_2 & D_2 & B_2\\ 0 & B_2 & 0\end{bmatrix}q' +\frac{H_3}{v_3}\bar{q}'\begin{bmatrix}0 & A_3 & 0\\ A_3 & 0 & B_3\\ 0 & B_3 & C_3\end{bmatrix}q' \tag{34}$$

and thus the mass matrix

$$M'=\frac{1}{\sqrt{2}}\begin{bmatrix}E_1 & \sum_i A_i & 0\\ \sum_i A_i & D_2 & \sum_i B_i\\ 0 & \sum_i B_i & C_3\end{bmatrix}. \tag{35}$$

B. The extended Fritzsch scheme

Although the cyclic Fritzsch ansatz treats the Higgs scalars more equally, it still clearly distinguishes between Higgs scalars and fermions. The extended Fritzsch ansatz simply states that $\lambda'_{ijk}\neq0$ when either $i=j=k$ or when any two are equal and differ from the third by one ($i=j=k\pm1$, $i=k=j\pm1$, $j=k=i\pm1$)

$$\frac{H_1}{v_1}\bar{q}'\begin{bmatrix}E_1^* & A_1 & 0\\ A_1 & D_1 & 0\\ 0 & 0 & 0\end{bmatrix}q'+\frac{H_2}{v_2}\bar{q}'\begin{bmatrix}E_2 & A_2 & 0\\ A_2 & D_2^* & B_2\\ 0 & B_2 & C_2\end{bmatrix}q' +\frac{H_3}{v_3}\bar{q}'\begin{bmatrix}0 & 0 & 0\\ 0 & D_3 & B_3\\ 0 & B_3 & C_3^*\end{bmatrix}q' \tag{36}$$

and the mass matrix M' is

$$M'=\frac{1}{\sqrt{2}}\begin{bmatrix}E_1^*+E_2 & A_1+A_2 & 0\\ A_1+A_2 & D_1+D_2^*+D_3 & B_2+B_3\\ 0 & B_2+B_3 & C_2+C_3^*\end{bmatrix}. \tag{37}$$

This ansatz should be of special interest to superstring devotees. If the heterotic superstring theory is compactified on an orbifold, the various **27**'s of E_6 may lie in different twisted sectors. As discussed in Ref. 25, the communication between sectors is exponentially suppressed. As a result, the Higgs field and fermions within a given **27** will have relatively large interactions, couplings between two fields in one **27** and one in another nearby sector will be smaller, and the other couplings will be much smaller still. Thus, the extended Fritzsch ansatz may arise naturally in a superstring compactification. Of course in such an orbifold compactification one would expect the Yukawa couplings in (34) marked with asterisks to have the largest values. In the following we shall, however, not impose this restriction and keep our discussion in terms of a more general range of parameters.

C. Masses and mixing angles

Neither of these two mass matrices (35) or (37) is Fritzsch type, and so we must consider the mixing-angle predictions for them, and then calculate the magnitude of the neutral flavor-changing couplings. Rather than consider each separately, we consider the general 3×3 symmetric matrix

$$M'=\begin{bmatrix}E & A & 0\\ A & -D & B\\ 0 & B & C\end{bmatrix}.$$

Without loss of generality, we take $C\gg D,E$ and we require that the hierarchy of eigenvalues does not arise through very delicate cancellations. If the eigenvalues are m_b, m_s, and m_d, it is straightforward to show that

$$\begin{aligned} &m_b\simeq C\ ,\\ &m_s\simeq\max(D,B^2/m_b)\ ,\\ &m_d\simeq\max(E,A^2/m_s)\ . \end{aligned} \tag{38}$$

As expected, this result says that the down (strange) quark gets its mass either through the diagonal element or through mixing with the strange quark (bottom quark). We can readily calculate the Cabibbo angle, and find that

$$\sin\theta_C \simeq \frac{A_d}{m_s} - \frac{A_u}{m_c} \;, \tag{39}$$

where $A_d(A_u)$ is the mass matrix A parameter for the $Q=-\frac{1}{3}$ $(\frac{2}{3})$ quarks. Since $A_d{}^2 \le m_d m_s$ and $A_u{}^2 \le m_u m_c$, the second term is much smaller than the observed $\sin\theta_C$ and thus $A_d \simeq m_s \sin\theta_C$. However, experimentally, $\sin\theta_C \simeq \sqrt{m_d/m_s}$, so $A_d{}^2 \simeq m_d m_s$ or $m_d \simeq A_d{}^2/m_s$. Thus $A^2/m_s \gg E$ in Eq. (38) (Ref. 26) and the down quark must get its mass through mixing with the strange quark. The only remaining option corresponds to choosing whether the strange quark gets its mass through a diagonal element or through mixing with the bottom quark. The two choices correspond to the mass matrices

$$M'_1 = \begin{bmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{bmatrix}, \quad M'_2 = \begin{bmatrix} 0 & A & 0 \\ A & -D & B \\ 0 & B & C \end{bmatrix} \;, \tag{40}$$

where, in M'_2, $B^2 \ll DC = m_s m_b$.

M'_1 is just the Fritzsch mass matrix, whose mixing-angle predictions were discussed in the last section. M'_2 is new. The orthogonal transformation that diagonalizes M'_2 is the matrix

$$O_2 \simeq \begin{bmatrix} 1 & \sqrt{m_1/m_2} & -(B/m_3)\sqrt{m_1/m_3} \\ -\sqrt{m_1/m_2} & 1 & -(B/m_3) \\ 0 & (B/m_3) & 1 \end{bmatrix} . \tag{41}$$

This yields the quark mixing elements as

$$\begin{aligned} V_{us} &\simeq V_{cd} \simeq \sqrt{m_d/m_s} \;, \\ V_{cb} &\simeq (B_d/m_b) - (B_u/m_t) \;, \\ V_{ub} &\simeq \sqrt{m_u/m_c}\,[(B_d/m_b) - (B_u/m_t)] \\ &= \sqrt{m_u/m_c}\, V_{cb} \;. \end{aligned} \tag{42}$$

The Cabibbo-angle result remains the same as the Fritzsch case, of course; and interestingly it also has the same result for the ratio (V_{ub}/V_{cb}) as the Fritzsch case, Eq. (28) (thus a specific prediction that is quite consistent with present experimental limit). The expression for V_{cb} is, however, different as $B_d(B_u)$ corresponds to the B parameter in the down (up) sector and is bound from above by the Fritzsch value of $\sqrt{m_s m_b}(\sqrt{m_c m_t})$. The significant point is that since in the M'_2 case B_d and B_u can be arbitrarily small, the KM element V_{cb} in Eq. (42) can easily be small enough without requiring m_t to be approximately equal to $m_c m_b/m_s$ as in the Fritzsch case. Hence, should m_t be greater than 80 GeV, the Fritzsch matrices will be disfavored and M'_2 remains a viable option.

Thus, we find that within the framework we are working and assuming no delicate cancellations, only two types of matrices give correct values for the quark mixing angles. One is the Fritzsch matrix M'_1. The other M'_2 predicts a small value for V_{cb} which, unlike M'_1, does not require a particular range of values for m_t.

D. Flavor-changing neutral currents

Even though the Yukawa coupling matrices λ'_{ijk} in the cyclic and extended cases (34) and (36) do not have the exact Fritzsch form, we shall demonstrate that for a broad range of parameters, we again recover the simple Fritzsch scheme result of $M^{(i)}_{jk} = \Delta_{jk}\sqrt{m_j m_k}$. Let us represent a generic coupling matrix in these two schemes as

$$(\lambda'_i)_{jk} = \frac{1}{v_i} \begin{bmatrix} \epsilon_i & \alpha_i & 0 \\ \alpha_i & \delta_i & \beta_i \\ 0 & \beta_i & \gamma_i \end{bmatrix} \tag{43}$$

and work out explicitly the similarity transformation by the orthogonal matrix O_2 of Eq. (41). Although this form is appropriate for the M'_2 mass matrix with $B \ll \sqrt{m_s m_b}$, the result can be easily converted to that for the M'_1 case by setting $B = \sqrt{m_s m_b}$. We shall not bother to write down all of the flavor-changing couplings, but will concentrate on the $\bar{d}s\, H_i$ couplings, corresponding to $l=1, n=2$ in Eq. (6):

$$\begin{aligned} M^{(i)}_{ds} \simeq{} & \alpha_i - 2\beta_i (B_d/m_b)\sqrt{m_d/m_s} + \gamma_i (B_d/m_b)^2 \sqrt{m_d/m_s} \\ & + (\delta_i - \epsilon_i)\sqrt{m_d/m_s} \;. \end{aligned} \tag{44}$$

The specific expressions for α, β, γ, δ, and ϵ, can be read off directly by comparing Eq. (43) with (34) and (36).

Again we shall assume that each Yukawa coupling matrix (λ') has the same hierarchical structure as the final mass matrix M'. Namely, in the sum $M'_{jk} = \sum_i v_i \lambda'_{ijk}/\sqrt{2}$ there are no delicate cancellations. Thus, for the cyclic case, for either M'_1 or M'_2, we have

$$\begin{aligned} & A_i \equiv a_i \sqrt{m_d m_s} \quad \text{with } a_1 + a_2 + a_3 = 1 \;, \\ & E_1 \simeq 0, \quad C_3 \simeq m_b \;, \end{aligned} \tag{45}$$

a_i are ~ 1—in fact all such parameters b_i and c_i in the expressions below will be assumed to be ~ 1. We can easily find from Eq. (44) that all the dimensional couplings $M^{(i)}_{ds}$ are proportional to $\sqrt{m_d m_s}$, with coefficients

$$\begin{aligned} & M^{(1)} \simeq a_1 - 2b_1, \quad M^{(2)} \simeq a_2 - 2b_2 \;, \\ & M^{(3)} \simeq a_3 - 2b_3 + 1 \end{aligned} \tag{46}$$

for the cyclic M'_1 case with $D_2 \simeq 0$ and $B_i = b_i \sqrt{m_s m_b}$ (thus $b_1 + b_2 + b_3 = 1$), and

$$M^{(1)} \simeq a_1, \quad M^{(2)} \simeq a_2 - 1, \quad M^{(3)} \simeq a_3 \tag{47}$$

for the cyclic M'_2 case with $D_2 \simeq m_s$ and $B_i = O(B_d) \ll \sqrt{m_s m_b}$.

Similarly, for the extended case[26]

$$\begin{aligned} & A_i \simeq a_i \sqrt{m_d m_s} \quad \text{with } a_1 + a_2 = 1 \;, \\ & C_i \simeq c_i m_b \quad \text{with } c_2 + c_3 = 1 \;, \\ & E_i \simeq 0 \;, \end{aligned} \tag{48}$$

we find the coupling $M^{(i)}_{ds}$ being proportional to $\sqrt{m_d m_s}$, with coefficients

$$M^{(1)} \simeq a_1, \quad M^{(2)} \simeq a_2 - 2b_2 + c_2, \quad M^{(3)} \simeq -2b_3 + c_3 \tag{49}$$

for the extended M_1' case with $B_i \simeq b_i\sqrt{m_s m_b}$ (thus $b_2+b_3=1$) and $D_i \simeq 0$; and

$$M^{(1)} \simeq a_1 - d_1,\ M^{(2)} \simeq a_2 - d_2,\ M^{(3)} \simeq -d_3 \tag{50}$$

for the extended M_2' case with $B_i = O(B_d) \ll \sqrt{m_s m_b}$ and $D_i \simeq d_i m_s$ (thus $d_1+d_2+d_3=1$).

In this section, we have presented two alternatives to the Fritzsch ansatz which are more appropriate to multiple-Higgs-scalar models, and shown that each can accommodate the observed masses and mixing angles. We have also shown that the flavor-changing neutral couplings will, if the products of the Yukawa couplings and expectation values of the Higgs fields are comparable, be $O(\sqrt{m_d m_s})$ and thus the lower bound on the Higgs-boson masses is ~ 1 TeV. It should be emphasized that we have neglected various mixing-angle factors, possible cancellations between Higgs-boson exchanges, etc., which could lower this bound significantly.

V. DISCUSSION AND SUMMARY

In this paper we have considered the aesthetically pleasing possibility that there is a multigenerational Higgs-boson structure which matches that of the fermions. (This is also a requirement in supersymmetric E_6 models.) However, the notion of multiple Higgs scalars runs counter to the prevailing theoretical opinion that such a Higgs-boson structure will bring about unacceptably large flavor-changing neutral-current effects (especially too large a kaon mass difference Δm_K) and the accompanying new source of *CP* violation (due to Higgs-boson exchange) may imply too large an ϵ'/ϵ ratio in the neutral-kaon system. In more quantitative terms it has been shown[9,13] that Δm_K places a lower limit of 150 TeV for the mass of the neutral Higgs particle with flavor-changing couplings. Such a result is based on the assumption that all Yukawa couplings are proportional to the heaviest fermion mass of the given charge; in particular, the relevant coupling for Δm_K is taken to be proportional to $M_{sd} \simeq m_b$. In this paper we have argued that Yukawa couplings, being closely related to the fermion mass matrices, are likely to have a structure reflecting the observed fermion mass hierarchy. Three examples of phenomenological Yukawa couplings schemes are presented in this paper. They yield either the Fritzsch mass matrix, in our notation M_1', or another type, the M_2'. In both cases we have the successful relation of Cabibbo angle $\theta_C \simeq \sqrt{m_d/m_s}$ and the prediction $\Gamma(b\to u e\nu)/\,\Gamma(b\to c e \nu) \simeq 0.006$. The mixing V_{cb} can be small (as experimentally observed) in the M_1' scheme if m_t lies in the range of 20–80 GeV, while the M_2' type allows for an arbitrarily small V_{cb} with no constraint on m_t. All three examples of Yukawa couplings have a hierarchical structure of $M_{jk} \sim \sqrt{m_j m_k}$, thus for Δm_K a coupling of $M_{sd} \sim \sqrt{m_s m_d}$. This lowers the m_H limit to ~ 1 TeV. If m_H is not significantly above this range we may encounter the Higgs flavor-changing effects in the extremely sensitive neutral K, B, D, and T systems competing with the standard gauge-boson contributions. The standard model predicts that Δm_D is particularly suppressed; here the Higgs-boson contribution may actually dominate, yielding a value comparable to the present experimental limit. Thus a discovery in the near future of an Δm_D value significantly greater than the standard-model prediction can be interpreted as indicating a multigenerational Higgs-boson structure as discussed here.

Finally we should remark that with $m_H \simeq 1$ TeV the resulting *CP* violation is of the superweak type since the Higgs effective four-fermion coupling, when compared to the Fermi strength, is

$$G_{\rm Higgs}/G_F \simeq (m_q/m_H)^2 \lesssim 10^{-12}\ .$$

Consequently this possible new source of *CP* violation makes a negligible contribution to the ϵ'/ϵ ratio in the neutral K system.[27]

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[2]D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985).

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[6]H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979); L. F. Li, Phys. Lett. **84B**, 461 (1979).

[7]See, for example, A. Davidson, V. P. Nair, and K. C. Wali, Phys. Rev. D **29**, 1513 (1984); M. Shin, Phys. Lett. **145B**, 285 (1984); T. P. Cheng and L. F. Li, Phys. Rev. D **34**, 219 (1986).

[8]We shall consider only real symmetric Yukawa coupling matrices; hence only orthogonal matrices $O_L = O_R = O$ for diagonalizing M'.

[9]See, for example, B. McWilliams and L. F. Li, Nucl. Phys. **B179**, 62 (1981); O. Shanker, *ibid.* **B206**, 253 (1982).

[10]We are ignoring the distinction between weak and mass eigenstates of the Higgs fields. This introduces an arbitrary mixing angle ~ 1 into each term, which will not affect our arguments. See also Eq. (13) with respect to the related matter of "characteristic Higgs-boson mass" m_H.

[11]B. McWilliams and O. Shanker, Phys. Rev. D **22**, 2853 (1980). For reference one may note that this result agrees well with that obtained with the method of inserting vacuum state in all possible ways:

$$\langle \bar K \mid (\bar s\gamma_5 d)(\bar s \gamma_5 d) \mid K\rangle_{\rm vac} = -\tfrac{1}{6}\langle \bar K \mid \bar s\gamma_\lambda\gamma_5 d \mid 0\rangle\langle 0 \mid \bar s \gamma^\lambda\gamma_5 d \mid K\rangle + \tfrac{11}{6}\langle \bar K \mid \bar s\gamma_5 d \mid 0\rangle\langle 0\mid \bar s\gamma_5 d \mid K\rangle = \frac{f_K{}^2 m_K{}^2}{12 m_K}\left[-1+\frac{11 m_K{}^2}{(m_s+m_d)^2}\right]$$

because $\langle \bar{K} \mid \bar{s}d \mid 0\rangle$ and $\langle \bar{K} \mid \bar{s}\gamma_\lambda d \mid 0\rangle$ vanish due to parity, and $\langle \bar{K} \mid \bar{s}\sigma_{\mu\lambda} d \mid 0\rangle = 0$ as we cannot construct an antisymmetric tensor out of the kaon momentum. We then have

$$\langle \bar{K} \mid (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \mid K\rangle_{\text{vac}} \simeq \frac{11}{12}\frac{f_k{}^2 m_k{}^2}{m_s{}^2}$$

$$\simeq 8.7\times 10^{-2}\ \text{GeV}^3 .$$

[12]Througout this paper we shall for definiteness use the quark mass values (at 1 GeV) estimated by H. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982): $m_b \simeq 5.3$ GeV, $m_c \simeq 1.35$ GeV, $m_s \simeq 175$ MeV, $m_d \simeq 8.9$ MeV, and $m_u \simeq 5.1$ MeV.

[13]O. Shanker in Ref. 9 has also estimated this m_H lower bound from μe conversion in nucleus, yielding $m_H > 11$ TeV; $K \rightarrow \mu e$ and $K \rightarrow ee$, 7 TeV; $K \rightarrow \mu\mu$, 4.7 TeV; $K \rightarrow \pi\mu e$, 0.7 TeV; $\mu \rightarrow 3e$, 0.4 TeV; $\mu \rightarrow e\gamma$, 0.2 TeV.

[14]B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D **16**, 1519 (1977).

[15]M. Drees, Phys. Rev. D **35**, 2910 (1987); H. Haber and M. Sher, *ibid.* **35**, 2206 (1987).

[16]A. Datta, Phys. Lett. **154B**, 287 (1985).

[17]For a recent review see R. H. Schindler, in Proceedings of the 23rd International Conference on High Energy Physics, Berkeley, California, 1986, edited by S. Loken (World Scientific, Singapore, to be published).

[18]If the coupling is given by $\Delta_{jk}\sqrt{m_j m_k}$, then this should be multiplied by Δ_{sd}/Δ_{cu}, which is expected to be ~ 1.

[19]Particle Data Group, Phys. Lett. **170B**, 1 (1986).

[20]T. P. Cheng and L. F. Li (Ref. 7). Also, in this paper all KM angles are worked out with "maximal *CP*" phases. However, the essential results for the magnitudes V_{us}, V_{cb}, and V_{ub} are the same as stated here with all mass matrix elements taken to be real.

[21]A. Chen *et al.*, Phys. Rev. Lett. **52**, 1082 (1984).

[22]We take the v_i's to be all comparable. In many supersymmetric E_6 models it is assumed that only one v_i is nonzero. This can only occur if there is no mixing between the Higgs fields. Although this might occur for topological reasons, in general mixings will exist and the v_i's are expected to be comparable.

[23]To extend this argument further, as noted in the previous footnote the Higgs-boson mass eigenstates H_i are some linear combinations of the weak eigenstates, $H_i = \sum_l U_{il} H_l'$; the couplings λ_{ijk}' in Eq. (1) are linear combinations of the Yukawa couplings λ_{ijk}'' defined with respect to the fermion *and* the Higgs weak eigenstates: $\lambda_{ijk}' = \sum_l U_{il}\lambda_{ljk}''$. Thus the mass matrix of Eq. (23) results from a double sum $M_{jk}' = \sum\sum_{il} v_i U_{il} \lambda_{ljk}''/\sqrt{2}$. If there are any zeros, they result most likely from some discrete symmetry and/or topological restrictions acting on weak eigenstates and first appear in λ_{ijk}''. In order for such zeros to survive all the way down to the mass matrix M', the most natural way is for them to have fixed locations throughout.

[24]If the v_i form a hierarchy, say $v_1 \gg v_2 \gg v_3$ then it is easy to show that the lower bound on the mass H_1 is decreased by $O(v_2/v_1)$.

[25]L. E. Ibañez, Phys. Lett. **181B**, 270 (1986); K. Ito, *ibid.* **184B**, 331 (1987).

[26]Given the previous remarks about the relative size of Yukawa couplings in an orbifold compactification, it can be realized only if v_1 is correspondingly small.

[27]The more familiar *CP*-violation mechanisms due to Higgs-boson exchanges is of the milliweak type since a value of $m_H \simeq 10-50$ GeV is taken with flavor-changing effects suppressed by specific coupling arrangements; see, for example, the model of S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).