Axial Anomaly and the Proton Spin

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Several authors have noted that the forward matrix element of the axial-vector current as measured in polarized deep-inelastic electroproduction represents not only the quark contribution to the target-proton spin but also the gluonic component due to the anomaly. We show that the anomalous divergence equations constrain the size of this gluonic contribution. In particular it is of such sign and magnitude that the resulting quark content of the proton spin deviates even further from the naive-quark-model expectations.

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There is evidence suggesting that the strange-quark content of the proton may be very significant: It has been known for a long time that the size of the σ term as extracted from πN scattering data appears to be much larger than a $\sigma_{\pi N}$ value deduced from the naive-quark-model (NQM) picture of the nucleon as consisting mainly of the up and down quarks.[1] More recently a straightforward interpretation of the result obtained by the European Muon Collaboration[2] (EMC) in a deep-inelastic scattering experiment using a polarized muon beam on a polarized target also seems to indicate a significant contribution to the target-proton spin by the strange-quark sea.[3,4] However, this second piece of evidence has been called into question when several authors[5–7] have pointed out that the axial-vector current matrix element measured in the electroproduction also includes the gluonic spin contribution due to the axial-vector anomaly.[8] There is thus the possibility that the apparent spin contribution due to the strange quark may actually be that of the gluon. In this paper we shall argue, by way of pole dominance of the octet axial-vector divergence, that this is most likely not the case. We compute the anomaly term by using the current divergence equations in which the flavor-singled pseudoscalar density-matrix element is given approximately by the correction to the Goldberger-Treiman relation.[9] The gluon contribution thus obtained seems to be of the wrong sign to reduce the strange-quark contribution.

The EMC result may be stated in terms of the first moment of the polarized proton structure function when a standard Regge behavior for small values of x is assumed[10]:

$$\int_0^1 dx\, g_1^p(x,Q^2) = 0.114 \pm 0.012 \pm 0.026\,, \tag{1}$$

for $\langle Q^2 \rangle = 10.7\ \text{GeV}^2$. This integral is related via operator-product expansion to the forward matrix element of the axial-vector current $(\sum_i e_i^2 \bar{q}_i \gamma_\mu \gamma_5 q_i)$ taken between the proton states.[11] Defining $\Delta q_i'$ as in

$$\Delta q_i' \bar{\Psi} \gamma_\mu \gamma_5 \Psi = \langle p \,|\, \bar{q}_i \gamma_\mu \gamma_5 q_i \,|\, p \rangle\,, \tag{2}$$

where Ψ is the proton wave function, and using the flavor-SU(3) relations[12]

$$\begin{aligned} &\Delta u' - \Delta d' = F + D = g_A = 1.254 \pm 0.006\,, \\ &\Delta u' + \Delta d' - 2\Delta s' = 3F - D = 0.685 \pm 0.08\,, \end{aligned} \tag{3}$$

one obtains, separately for each flavor, the quark-spin contributions[13]:

$$\begin{aligned} &\Delta u' = 0.74 \pm 0.08, \quad \Delta d' = -0.51 \pm 0.08\,, \\ &\Delta s' = -0.23 \pm 0.08\,. \end{aligned} \tag{4}$$

These numbers are surprising from the viewpoint of the simple quark model, which would lead one to expect a very small strange-quark content and, certainly, a vanishingly small strange-quark contribution to the proton spin.[14] Furthermore, the three quark-spin terms of Eq. (4) sum up to zero[4] (with large errors),

$$\sum_i \Delta q_i' \equiv \Delta u' + \Delta d' + \Delta s' = 0.00 \pm 0.24\,, \tag{5}$$

leading to the apparent conclusion that the net proton spin is not all carried by its component quarks.

This simple interpretation has been challenged by several authors. Efremov and Teryaev,[5] Altarelli and Ross,[6] and Carlitz, Collins, and Mueller[7] have pointed out that due to the Adler-Bell-Jackiw axial-vector anomaly the matrix element $\langle p \,|\, \bar{q}_i \gamma_\mu \gamma_5 q_i \,|\, p \rangle$ in Eq. (2) in fact measures the linear combination of the helicity component of the quark Δq, and that of the gluon Δg,

$$\Delta q' = \Delta q - (\alpha_s / 2\pi) \Delta g\,. \tag{6}$$

In this equation the gluon helicity component Δg is defined by

$$-\Delta g\, 2M \bar{\Psi} i \gamma_5 \Psi = \langle p \,|\, \mathrm{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} \,|\, p \rangle\,, \tag{7}$$

where M is the proton mass, $G_{\mu\nu}$ is the gluon field intensity, and $\tilde{G}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}$. One can heuristically understand this by recalling that the axial-vector U(1) current $A_\mu=\sum_i^{N_f}\bar{q}_i\gamma_\mu\gamma_5 q_i$ has the anomalous divergence $N_f(\alpha_s/2\pi)\mathrm{Tr}G\tilde{G}$. Consequently, we can regard it as being made up of two parts one being a quark current $\tilde{A}_\mu$ which is conserved in the massless quark limit and the other being a gluonic current K_μ which has the anomaly as its divergence:

$$A_\mu=\tilde{A}_\mu+K_\mu\,,$$

with

$$\partial^\mu K_\mu=N_f(\alpha_s/2\pi)\mathrm{Tr}G\tilde{G}\,. \tag{8}$$

Therefore, the matrix element of the usual axial-vector current $\bar{q}_i\gamma_\mu\gamma_5 q_i$ contains a quark and a gluonic term as in Eq. (6). With this realization it appears that the knowledge of the EMC result is not enough to ascertain the quark-spin contributions Δq_i. Thus, such questions as the validity of the original expectation that Δs should be small remain unanswered.

Since Δg in Eq. (7) is simply the anomalous divergence of the axial-vector current, it is more useful to work with the current divergence equations,

$$\partial^\mu(\bar{q}_i\gamma_\mu\gamma_5 q_i)=2m_i\bar{q}_i i\gamma_5 q_i+\frac{\alpha_s}{2\pi}\mathrm{Tr}G\tilde{G}\,, \tag{9}$$

for each flavor of $q_i=u,d,s,\ldots$. Taking the matrix elements of these equations between the proton states with zero momentum transfer ($Q^2=0$), we get for the case of three flavors,

$$2M\Delta u'=2m_u v_u-2M\left(\frac{\alpha_s}{2\pi}\Delta g\right), \tag{10a}$$

$$2M\Delta d'=2m_d v_d-2M\left(\frac{\alpha_s}{2\pi}\Delta g\right), \tag{10b}$$

$$2M\Delta s'=2m_s v_s-2M\left(\frac{\alpha_s}{2\pi}\Delta g\right), \tag{10c}$$

where

$$v_q\bar{\Psi}i\gamma_5\Psi=\langle p\,|\,\bar{q}i\gamma_5 q\,|\,p\rangle\,. \tag{11}$$

In order to solve for the gluonic contribution Δg from the knowledge of $\Delta q'$ given in Eq. (4), we must have some handle on the matrix elements of the pseudoscalar density $v_q s$. It turns out that we can estimate the combination v_u+v_d by the usual method of Goldstone-pole saturation of the axial-vector-current divergence as in the derivation of the Goldberger-Treiman relation.

Recall that the divergence equation for the charged axial-vector current is given by

$$\partial^\mu\bar{u}\gamma_\mu\gamma_5 d=(m_u+m_d)\bar{u}i\gamma_5 d\,. \tag{12}$$

When this equation is taken between the nucleon states, the left-hand side yields $2Mg_A$ and on the right-hand side the pseudoscalar matrix element is approximated by its Goldstone pion π^+ contribution[15] yielding $2f_\pi g_{\pi NN}$, and hence the Goldberger-Treiman relation.

Now consider the divergence equation for the neutral component of the isovector axial-vector current,

$$\partial^\mu(\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d)=2m_u\bar{u}i\gamma_5 u-2m_d\bar{d}i\gamma_5 d=(m_u+m_d)(\bar{u}i\gamma_5 u-\bar{d}i\gamma_5 d)+(m_u-m_d)(\bar{u}i\gamma_5 u+\bar{d}i\gamma_5 d)\,. \tag{13}$$

Taking the matrix element of this equation between the proton states with zero momentum transfer ($Q^2=0$), we get

$$2Mg_A=(m_u+m_d)(v_u-v_d)+(m_u-m_d)(v_u+v_d)\,. \tag{14}$$

The matrix element of the isovector density in the first term on the right-hand side is dominated by the π^0 pole. We then get for the isosinglet matrix element in the second term

$$v_u+v_d=2M\,\delta g_A/(m_u-m_d)\,, \tag{15}$$

where δg_A is the correction to the Goldberger-Treiman expression for g_A:

$$\delta g_A=g_A-\frac{f_\pi}{M}g_{\pi NN}=-0.077\,. \tag{16}$$

This information on v_u+v_d, which is obtained *without any (pole-dominance) approximation in the singlet channel itself,* allows us now to solve for the gluonic matrix element from Eq. (10):

$$-\frac{\alpha_s}{2\pi}\Delta g=\frac{\Delta u'}{1+z}+\frac{z\Delta d'}{1+z}-\frac{2z\,\delta g_A}{z^2-1}\,, \tag{17}$$

where[16] $z=m_u/m_d=0.56$. Or combining Eqs. (3) and (5) to get

$$\begin{aligned}\frac{\alpha_s}{2\pi}\Delta g&=\left(\frac{\frac{2}{3}}{1+F/D}-\frac{1}{1+z}\right)g_A+\frac{2z}{z^2-1}\delta g_A-\tfrac{1}{3}\sum_i\Delta q_i'\\&=-0.29+0.13\pm0.08\\&=-0.16\pm0.08\,.\end{aligned} \tag{18}$$

This is our principal result, of which some comments are in order.

(1) *Sign of the gluonic component Δg and comparison with NQM.*—The simple quark model would lead us to expect $\Delta s\cong 0$ and $\sum_i\Delta q_i\cong 1$. In fact the gluonic term in Eq. (18) is such as to yield quark-spin distributions that deviate even further from these expectations as compared to those $\Delta q_i'$ in Eqs. (4) and (5):

$$\Delta u=0.58,\quad \Delta d=-0.67,\quad \Delta s=-0.39$$

and

$$\sum_i\Delta q_i=-0.48\,. \tag{19}$$

It is clear that we have just the "wrong" sign for the gluonic term from the viewpoint of the NQM. The only part of our calculation that is possibly somewhat uncertain concerns the approximation of the isosinglet density v_u+v_d in terms of δg_A. One expects that there will be corrections to the pole dominance in the derivation of the Goldberger-Treiman relation from the charged divergence Eq. (12),

$$2Mg_A=2f_\pi g_{\pi NN}+\mu_+$$

and from the neutral divergence Eq. (14),

$$2Mg_A=2f_\pi g_{\pi NN}+\mu_0+(m_u-m_d)(v_u+v_d)\,.$$

Thus,

$$(m_u-m_d)(v_u+v_d)=\mu_+-\mu_0\,, \tag{20}$$

where μ_+ and μ_0 are the corrections to the pole dominance. Generally μ_+ and μ_0 are different. If we assume that corrections to the pole dominance are somewhat isospin independent, i.e., $\mu_+=\mu_0$, we will get[17]

$$v_u+v_d=0\,, \tag{21}$$

which yields through Eq. (18)

$$\frac{\alpha_s}{2\pi}\Delta g=-0.29\pm0.08\,. \tag{22}$$

This changes the previous result in the wrong direction to accommodate the NQM expectation. In fact it is straightforward to see that in order to change the sign of Δg we must have a singlet matrix element $v_\mu+v_d$ comparable in magnitude to that of v_u-v_d.[18] From Eq. (20) we see that the correction to the Goldberger-Treiman relation (μ_+) has two components: μ_0 is the correction to the π^0 pole dominance [i.e., π^0 PCAC (partial conservation of axial-vector current)] and the other component is proportional to v_u+v_d. Thus a large value of v_u+v_d means a large value of μ_0 and they must somehow cancel each other to get a small μ_+. This does not seem to be likely. Furthermore it contradicts our experience with the successes of PCAC for the neutral-pion system (e.g., $\pi^0\to\gamma\gamma$). Thus μ_+ should typify the size of μ_0 and $(m_u-m_d)(v_u+v_d)$ terms. This is the basis of our argument that the estimate of v_u+v_d by δg_A as given in Eq. (15) is reliable, and the sign of Δg as given in Eq. (18) is correct.

What kind of phenomenological test of this sign do we have? It has already been pointed out by Carlitz, Collins, and Mueller[7] that the dominant source of polarization-dependent high-k_T jets in deep-inelastic scattering is this gluonic term. Thereby, if the gluonic component is indeed negative, the structure function $g_1(x)$ calculated from scattering events with a moderate k_T cutoff will be found to have a first moment that deviates even further from the Ellis-Jaffe sum rule[14] than Eq. (1).

(2) *Magnitude of* Δg.— Clearly the value of gluonic component Δg will depend on Q^2. However, the product $\alpha_s\Delta g$ is Q^2 independent.[6,7] Our results derived at $Q^2=0$ suggests that even for a moderate value of $Q^2\simeq 10\ \text{GeV}^2$ corresponding to $\alpha_s\cong0.2$ there is already a fairly sizable $\Delta g\cong -5$. The phenomenological implication for such a gluonic component has already been discussed in Ref. 7. This large negative quantity which presumably must be canceled by the orbital angular momentum contributions is by itself not necessarily so bizarre if one recalls that even for a positive Δg its growth at high Q^2 will have to be counterbalanced by a large negative orbital component.

(3) *Pseudoscalar matrix elements.*— While the products of $m_q v_q s$ as derived from Eqs. (10) and (18) are quite comparable in magnitudes[19]

$$m_u v_u=545\ \text{MeV},\quad m_d v_d=-630\ \text{MeV},\qquad m_s v_s=-367\ \text{MeV}\,, \tag{23}$$

which are related to the pseudoscalar Higgs-boson–nucleon couplings, the strange pseudoscalar density-matrix element is very much suppressed:

$$\frac{\langle p\,|\,\bar s i\gamma_5 s\,|\,p\rangle}{\frac12\langle p\,|\,\bar u i\gamma_5 u-\bar d i\gamma_5 d\,|\,p\rangle}\cong0.02\,. \tag{24}$$

This is to be compared to the scalar density ratio as deduced from $\sigma_{\pi N}$[1]:

$$\frac{\langle p\,|\,\bar s s\,|\,p\rangle}{\frac12\langle p\,|\,\bar u u+\bar d d\,|\,p\rangle}\cong0.47\,. \tag{25}$$

However, we should note that this is not a manifestation of the Okubo-Zweig-Iizuka rule because Eq. (24) is really a result of the smallness of v_u+v_d as well as $v_u+v_d+v_s$, reflecting the absence of Goldstone pole in these singlet channels. In fact our derivation makes it clear that the matrix element will still be suppressed even when the pseudoscalar density $\bar s i\gamma_5 s$ is sandwiched between strange baryon states.

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[18]For the NQM value of $\Delta s = 0$, i.e., $(\alpha_s/2\pi)\Delta g = +0.23$, we obtain the ratio $r \equiv (\nu_u + \nu_d)/(\nu_u - \nu_d) \geq 0.7$ compared to $r \cong 0.2$ for our result of Eq. (18). Ellis and Karliner in Ref. 10 obtained $\Delta g = 0$ in a particular version of the Skyrme model and thus a value of r about midway in between.

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