# Effects of superheavy neutrinos in low-energy weak processes

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(Received 31 May 1991)

We examine the validity of the decoupling theorem for a superheavy neutrino in the standard model augmented, in one case, by a sequential fourth generation of lepton doublet, and, in another, by a singlet neutrino field. We present a detailed $R_\xi$ gauge calculation of the $\mu \rightarrow e\gamma$ decay amplitude with its exact dependence on the neutrino mass $m$. Our calculation shows clearly that the resultant amplitude is nonvanishing in the large-$m$ limit because of the contribution coming from the longitudinal gauge bosons. In such diagrams the usual decoupling theorem is invalidated as the Yukawa coupling grows with $m$. Other higher-order processes such as $\mu \rightarrow 3e$, $\mu e$ conversion in the nucleus, etc., as well as the $\rho$ parameter, are also studied. The large-$m$ behavior of various amplitudes are summarized in a table. If the source of the large mass does not involve a large coupling constant, as is the case in the seesaw model for neutrino masses, then the decoupling property is recovered when the mass dependence of the heavy-light mixing angle is taken into account. In several cases, notably the induced $\mu e Z$ vertex and the $\rho$ parameter, this recovery of the decoupling suppression involves some complicated cancellations.

## I. INTRODUCTION

In recent years the emphases of particle-physics phenomenology have been on the test of the standard model (SM), and on the search for new physics beyond the electroweak scale. In addition to direct searches, an important approach has been the precision measurements that are sensitive to virtual effects of such heavy particles [1]. For example, a bound on the top-quark mass can be deduced from the study of radiative corrections. One of the key features that makes this possible is the violation of the decoupling theorem. Ordinarily we expect heavy particles to decouple in low-energy processes. Namely, the effects of heavy particles in the virtual intermediate states are suppressed by inverse powers of the heavy-particle mass. Physically this seems reasonable: the study of any physical phenomena at a given distance scale should not depend sensitively on our knowledge of the physics on much shorter scales. The proof of this decoupling theorem has been given by Applequist and Carazzone [2]. However, their proof is valid only for the unbroken gauge theories, and instances of nondecoupling of heavy particles in theories with spontaneous symmetry breaking have been noted [3]. Nondecoupling is very interesting because it allows exploring the physics at a high-energy scale through low-energy processes. Because of its importance, we would like to have a better understanding of how and where such nondecoupling can occur. In this paper we will provide further details of the mechanism by which heavy particles evade the decoupling theorem, and of the mechanism by which heavy particles which are *singlets* with respect to the electroweak group recover the decoupling property in the end.

In addition to the problem of a general understanding of the decoupling theorem, we are interested in specific instances where heavy particles yield definite corrections to the SM, particularly those by heavy fermions. In this connection, we have in mind the top quark, technicolor and supersymmetric fermions, and heavy leptons. The importance of calculating the $t$-quark radiative effects has long been recognized: as a guide to its eventual discovery (by providing bounds on the top-quark mass), and as the important background that must be subtracted out when searching for other new physics. Recently there has been an upsurge of interest to calculate and classify the radiative effects of the technicolor and supersymmetric particles [4]. Here we shall concentrate on the discussion of "heavy neutrinos." (Our results can obviously be transferred to superheavy quarks.) By heavy neutrinos we mean neutral, weakly interacting fermions with masses much greater than the $W$ and $Z$ gauge bosons. What we have in mind are two categories of models, which we shall call (a) sequential doublet neutrino models, and (b) seesaw singlet neutrino models.

*(a) Sequential doublet neutrino models.* These models are straightforward extensions of SM to more than three generations of quarks and leptons. The SLAC Linear Collider (SLC) and CERN $e^+e^-$ collider LEP results of course have excluded models with more than three light neutrinos [5]. But there is always the possibility that there are lepton doublets having heavy neutrinos with masses $m_\nu > 45$ GeV which could not be pair produced in $Z$ decays. Such a sequential scenario is conceptually very simple; it is worthwhile to consider some of its consequences.

For definiteness, we shall discuss a model with a fourth generation of a heavy lepton doublet. We denote the neutrino flavors as $\nu_a$ with $a = e, \mu, \tau, L$ and the neutrino mass eigenstates as $\nu_1$, $\nu_2$, $\nu_3$, and $\nu_4$. They are related through a unitary mixing matrix:

$$|\nu_a\rangle = \sum_{i=1}^{4} U_{ai}|\nu_i\rangle \ . \tag{1}$$

We shall assume that $m_{1,2,3}$ are small while $m_4$ is large, perhaps much greater than the electroweak scale.

*(b) Seesaw singlet neutrino models.* Here we have right-handed neutrino states, which are singlets under the electroweak group. Thus Majorana mass terms bilinear in such right-handed fields as well as the usual Dirac mass terms connecting the right- and left-handed neutrinos are present. Such Majorana terms, which can result from the vacuum expectation value of a singlet Higgs field or direct bare mass terms, are expected to have a magnitude (call it $M$) much larger than the electroweak scale, while the Dirac mass terms should be comparable to the familiar fermions (call its scale $\mu$). The resultant mass eigenstates comprise two classes of Majorana particles, one being heavy $O(M)$, one being superlight $O(\mu^2/M)$. This is the well-known seesaw mechanism for generating superlight neutrino masses [6]. In such models every left-handed neutrino state of a given flavor is an admixture of these light- and heavy-neutrino mass eigenstates with very small $O(\mu/M)$ mixing coefficients for the heavy components.

Again for definiteness we will consider a simple version of such models where only one right-handed singlet field is present. The three flavors of the left-handed neutrino states $\nu_a$ with $a=e,\mu,\tau$ are superpositions of four Majorana mass eigenstates: three states $\nu_{1,2,3}$ being superlight (in fact because the symmetric $4\times 4$ mass matrix has rank 2, two states $\nu_{1,2}$ are actually massless) and one heavy state $\nu_4$. Thus the pattern of mixing is somewhat similar to (1) except that the mixings of the heavy neutrinos in the left-handed flavor states are strongly constrained. Details of this model have been studied by Jarlskog [7]. Here we shall display $U_{ai}$ in terms of her parametrization:

$$\begin{aligned}
|\nu_e\rangle &= c_\beta|\nu_1\rangle + c_\alpha s_\beta|\nu_3\rangle + s_\alpha s_\beta|\nu_4\rangle \ ,\\
|\nu_\mu\rangle &= -s_\beta s_\gamma|\nu_1\rangle + c_\gamma|\nu_2\rangle\\
&\quad + c_\alpha c_\beta s_\gamma|\nu_3\rangle + s_\alpha c_\beta s_\gamma|\nu_4\rangle \ ,\\
|\nu_\tau\rangle &= -s_\beta c_\gamma|\nu_1\rangle - s_\gamma|\nu_2\rangle\\
&\quad + c_\alpha c_\beta c_\gamma|\nu_3\rangle + s_\alpha c_\beta c_\gamma|\nu_4\rangle \ ,
\end{aligned} \tag{2a}$$

where the mixing angles $\beta$ and $\gamma$ (there are altogether three angles in this $4\times 4$ orthogonal matrix because of the $\nu_1$-$\nu_2$ degeneracy) are related to ratios of Dirac mass terms and are *a priori* unrestricted, but the mixing of heavy component is given by

$$s_\alpha \equiv \sin\alpha = \left[\frac{m_3}{m_3+m_4}\right]^{1/2} , \tag{2b}$$

which is of order $\mu/M$ because $m_3 \simeq \mu^2/M$ and $m_4 \simeq M$.

The basic difference of the heavy neutrinos in these two models is that in the sequential model (a) the neutrino state has a nontrivial $SU(2)\times U(1)$ quantum number and a bare mass term is not compatible with the electroweak gauge symmetry. Thus the neutrino mass term comes from spontaneous symmetry breaking, and is proportional to the Yukawa coupling constant. On the other hand, the dominant neutrino mass term in the seesaw model (b) is gauge invariant by itself. Hence, in (b) the large mass limit does not entail a large Yukawa coupling. This basic difference accounts for the decoupling of the seesaw neutrino but not the sequential doublet $\nu$. Although the underlying mechanism is not difficult to understand, to see how important such nondecoupling effects are in any specific process and to see how decoupling is restored through, as we shall see, a rather intricate interplay of the heavy-light mixings, we must carry out explicit calculations.

In Sec. II we present a detailed one-loop calculation of the $\mu\to e\gamma$ amplitude without making the usual assumption of the intermediate neutral lepton mass being small (compared to gauge-boson masses). Furthermore we do the calculation in the general $R_\xi$ gauge so as to display the important role played by the unphysical Higgs boson (i.e., longitudinal gauge bosons) in avoiding the usual decoupling theorem. Other higher-order electroweak processes where a superheavy neutrino can be potentially important are also studied: In Sec. III we discuss $\mu\to 3e$ and $\mu e$ conversion in a nucleus and other muon-number-changing processes; in Sec. IV the question of coupling versus nondecoupling in the higher-order correction to the $\rho$ parameter will be discussed.

To fix the normalization of the relevant amplitudes, let us write down the effective Lagrangian for these low-energy weak processes [8]. We can first concentrate on the $\mu e\gamma$ and $\mu 3e$ processes:

$$L_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left[\frac{em_\mu}{(4\pi)^2}T(x)\bar{e}_L\sigma_{\lambda\nu}\mu_R F^{\lambda\nu} + \frac{\alpha m_\mu^2}{4\pi}E(x)\bar{e}_L\gamma_\lambda\mu_L(\bar{e}_L\gamma^\lambda e_L) + \frac{\alpha m_\mu^2}{4\pi\sin^2\theta_W}[F(x)\bar{e}_L\gamma_\lambda\mu_L + m_\mu G(x)\Box^{-1}\partial^\nu\bar{e}_L\sigma_{\lambda\nu}\mu_R](\bar{e}\gamma^\lambda e)\right] + \text{H.c.} \ , \tag{3}$$

where $x\equiv m^2/M_W^2$ with $m$ being the neutrino mass. $G_F$ is the Fermi constant, $\theta_W$ the weak mixing angle, $F^{\lambda\mu}$ the electromagnetic field tensor. The functions $T(x)$, $E(x)$, $F(x)$, and $G(x)$ are dimensionless. $T$ is the $\mu e\gamma$ amplitude, and $E$, $F$, and $G$ are those for $\mu 3e$. The semileptonic processes such as $\mu e$ conversion in the nucleus, $K_L\to\mu e$ decays, etc., can also be described by an effective Lagrangian with some of the leptonic bilinears replaced by appropriate quark currents. For all these processes the exact $x$-dependent results can be read off from the paper of Inami and Lim [8]. We will only study its large-$x$ limit, and see how the decoupling results can be recovered for the singlet case.

In Sec. V we discuss and summarize our results for the various processes in a table.

## II. $\mu \rightarrow e\gamma$

The most general gauge-invariant amplitude for the decay $\mu \rightarrow e + \gamma$, where the photon is on shell ($q^2=0$), has the form

$$M(\mu \rightarrow e\gamma) = i\epsilon^{\lambda}\bar{u}_e q^{\nu}\sigma_{\lambda\nu}(a+b\gamma_5)u_{\mu} ,$$

where $\epsilon^{\lambda}(q)$ is the photon polarization, $a$ and $b$ are the invariant amplitudes. In the approximation of $m_e=0$, we have the decay rate

$$\Gamma(\mu \rightarrow e\gamma) = \frac{m_{\mu}^3}{8\pi}(|a|^2+|b|^2) .$$

In the SM, these amplitudes are related to that in the effective Lagrangian (3) as

$$a = -b = \frac{G_F}{\sqrt{2}}\frac{em_{\mu}}{(4\pi)^2}T .$$

As the $\mu e\gamma$ amplitude (the first term in $L_{\text{eff}}$) corresponds to a dimension-five operator it must be represented by a set of loop diagrams. They must be finite contributions since there can be no renormalizable counterterms to absorb the infinities. The lowest-order one-loop diagrams (Fig. 1) naturally divide into two classes: where the photon line is attached onto external charged lines ($\mu$ or $e$) or onto internal lines. Since the former class can only yield amplitudes of the charge form factor type $\gamma_{\lambda}$, one only needs to concentrate on the second class, to get the magnetic-moment form-factor-type terms $\sigma_{\lambda\nu}$.

In this paper we are interested in the $\mu e\gamma$ transition as mediated by intermixing neutrinos, in particular, the exact neutrino mass $m$ dependence of the $\mu \rightarrow e\gamma$ amplitudes. Since the issue involves a possible violation of the decoupling theorem in a theory with spontaneous symmetry breaking, where the large fermion mass limit corresponds to the large Yukawa coupling limit, it will be illuminating to perform the calculation in the general $R_{\xi}$ gauge where the unphysical Higgs boson $\phi$'s are not explicitly eliminated. In this general class of gauge choices, the $W$ and $\phi$ propagators have the respective forms as

$$\Delta_{\mu\nu}(k,\xi) = -\frac{i}{k^2-M_W^2}\left[g_{\mu\nu}-\frac{k_{\mu}k_{\nu}}{k^2-M_W^2\xi^{-1}}\right] \qquad (4)$$

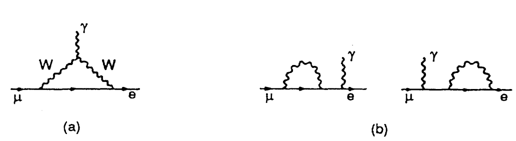



FIG. 1. One-loop diagrams for $\mu \rightarrow e\gamma$ via intermixing neutrinos.

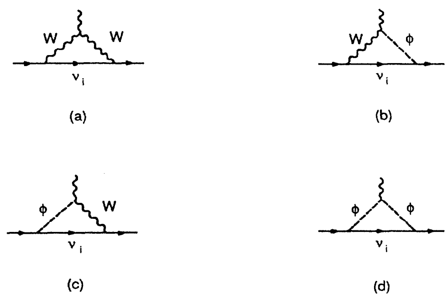



FIG. 2. (a) $WW$, (b) $W\phi$, (c) $\phi W$, and (d) $\phi\phi$ one-loop contributions to $\mu \rightarrow e\gamma$.

and

$$\Delta(k,\xi) = \frac{i}{k^2-M_W^2\xi^{-1}} .$$

The results [9] for the four diagrams (Fig. 2), modulo a common mixing angle $U_{\mu i}^* U_{ei}$, are

$$\begin{aligned} T_{WW} = {} & I_3(x)+I_2(x) \\ & +\frac{\xi-1}{\xi}\int_0^1 [\tfrac{3}{2}K_2(\alpha,x) \\ & \qquad +(\tfrac{3}{2}\alpha^2+\tfrac{1}{2}\alpha-\tfrac{3}{2})K_1(\alpha,x)]d\alpha , \end{aligned} \qquad (5a)$$

$$\begin{aligned} T_{W\phi} = {} & \int_0^1 K_1(\alpha,x)d\alpha - \frac{1}{2}\frac{\xi-1}{\xi}\int_0^1 (1-3\alpha) \\ & \times[(1-\alpha)K_0(\alpha,x)-K_1(\alpha,x)]d\alpha \\ & +\frac{x}{2}\int_0^1 [\alpha(\alpha-1)K_0(\alpha,x)+\alpha K_1(\alpha,x)]d\alpha \\ & +\frac{x\xi}{4}\int_0^1 \frac{\alpha^2(1-\alpha)}{\alpha+(1-\alpha)x\xi}d\alpha , \end{aligned} \qquad (5b)$$

$$\begin{aligned} T_{\phi W} = {} & \frac{x}{2}\int_0^1 \alpha[K_0(\alpha,x)-K_1(\alpha,x)]d\alpha \\ & -\frac{x\xi}{4}\int_0^1 \frac{\alpha(1-\alpha)(2-\alpha)}{\alpha+(1-\alpha)x\xi}d\alpha , \end{aligned} \qquad (5c)$$

$$T_{\phi\phi} = \frac{x\xi}{2}\int_0^1 \frac{\alpha(1-\alpha)(2-\alpha)}{\alpha+(1-\alpha)x\xi}d\alpha , \qquad (5d)$$

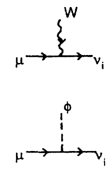
where

$$I_n(x) \equiv \int_0^1 \frac{\alpha^n}{\alpha+(1-\alpha)x}d\alpha , \qquad (6)$$

$$K_n(\alpha,x) \equiv \int_0^{1-\alpha} \frac{\beta^n}{(1-\alpha-\beta)\xi^{-1}+\beta+\alpha x}d\beta . \qquad (7)$$

The $W$ and $\phi$ couplings to the fermions are displayed in Fig. 3. Physically the unphysical Higgs boson is simply the longitudinal gauge boson. We note that its Yukawa coupling has a term proportional to the heavy fermion mass.

As a check with the previously known result [10] (and as a simple way to see the cancellation of the $\xi$ dependence), we take the small mass limit $x \rightarrow 0$. By keeping




$$\frac{ig}{2\sqrt{2}}\gamma_\lambda(1-\gamma_5)U_{\mu i}$$

$$-\frac{ig}{2\sqrt{2}M_W}[m_i(1-\gamma_5)-m_\mu(1+\gamma_5)]U_{\mu i}$$

FIG. 3. The $W$ and the unphysical-Higgs-boson couplings to the heavy neutrino $\nu_i$.

terms up to $O(x)$, we obtain

$$T_{WW}\to\tfrac{5}{6}-\tfrac{1}{2}g(\xi)-\frac{x}{4}[1-\tfrac{1}{3}f(\xi)+2g(\xi)]\ , \tag{8a}$$

$$T_{W\phi}\to\tfrac{1}{2}g(\xi)-\frac{x}{4}\left[\frac{5\xi}{6}+\frac{4}{3}f(\xi)-\frac{7}{3}g(\xi)\right]\ , \tag{8b}$$

$$T_{\phi W}\to-\frac{x}{4}\left[\frac{5\xi}{6}-f(\xi)+\tfrac{1}{3}g(\xi)\right]\ , \tag{8c}$$

$$T_{\phi\phi}\to\frac{x}{4}\frac{5\xi}{3}\ , \tag{8d}$$

where

$$f(\xi)\equiv\frac{\xi\ln\xi}{\xi-1},\quad g(\xi)\equiv\frac{\xi}{\xi-1}\left[1+\frac{\ln\xi}{\xi-1}\right]\ .$$

Clearly the sum is independent of $\xi$ and agrees with the well-known small mass limit result [11]:

$$T=\tfrac{5}{6}-\tfrac{1}{4}x\ . \tag{8e}$$

For the result in (5) which is exact in $x$, we note that in the 't Hooft gauge ($\xi=1$), the amplitudes can be computed easily and are

$$\begin{aligned}&T_{WW}=I_3(x)+\tfrac{1}{2}I_2(x),\quad T_{W\phi}=\tfrac{1}{2}I_2(x),\quad T_{\phi W}=0\ ,\\&T_{\phi\phi}=\frac{x}{2}[2I_1(x)-3I_2(x)+I_3(x)]\ .\end{aligned} \tag{9}$$

In the unitary gauge ($\xi=0$) the unphysical Higgs bosons $\phi$'s are not present, and correspondingly the $T_{WW}$ amplitude is equal to the sum of the amplitudes in (9):

$$T(x)=I_3+I_2+\frac{x}{2}(I_3-3I_2+I_1)\ . \tag{10}$$

This can be combined into a simple expression as

$$T(x)=\tfrac{1}{3}+\tfrac{3}{2}\int_0^1\frac{\alpha^3 d\alpha}{\alpha+(1-\alpha)x}\ . \tag{11}$$

Now let us examine the large mass limit $x\to\infty$. From the result of (11) it is clear that $T$ does not vanish [12], resulting in violation of the decoupling theorem:

$$T\to\tfrac{1}{3}\ . \tag{12}$$

To understand better the origin of this nondecoupling result, let us examine the limit of the individual terms in (5):

$$\begin{aligned}&T_{WW}\to 0\ ,\\&T_{W\phi}\to\tfrac{1}{4}\int_0^1\alpha^2 d\alpha+\frac{x}{2}\int_0^1[\alpha(\alpha-1)K_0+\alpha K_1]d\alpha\\&\quad=\tfrac{1}{12}-\tfrac{1}{4}\tfrac{1}{3}=0\ ,\\&T_{\phi W}\to-\tfrac{1}{4}\int_0^1\alpha(2-\alpha)d\alpha+\frac{x}{2}\int_0^1\alpha(K_0-K_1)d\alpha\\&\quad=-\tfrac{1}{6}+\tfrac{1}{6}=0\ ,\\&T_{\phi\phi}\to\tfrac{1}{2}\int_0^1\alpha(2-\alpha)d\alpha=\tfrac{1}{3}\ .\end{aligned} \tag{13}$$

Here we have used the limits of

$$I_n(x)\to 0\quad\text{and}\quad K_n(\alpha,x)\to\frac{(1-\alpha)^{n+1}}{\alpha(n+1)x}\ .$$

Thus the $2W$ amplitude vanishes, as to be expected from the decoupling theorem, while the other amplitudes which involve at least one unphysical Higgs line do not (although the constants in the limits of $T_{W\phi}$ and $T_{\phi W}$ happen to cancel). To understand the result, we note that the heavy fermion mass $m$ appears in the denominator of the propagator and in the numerator as the Yukawa coupling of the unphysical Higgs scalar. Thus in the large-$m$ limit, the important contributions are those diagrams containing unphysical Higgs bosons. It is also important to take into account the helicity structure to see whether the heavy fermion propagator gives $m^{-1}$ or $m^{-2}$ factors. We give the results for each graph below:

$$\begin{aligned}T_{WW}&\to\bar{u}_e\gamma_\lambda(1-\gamma_5)\left[\frac{\not p+\not k+m}{m^2}\right]\gamma_\nu(1-\gamma_5)u_\mu\\&=\bar{u}_e\gamma_\lambda\left[\frac{\not p+\not k}{m^2}\right]\gamma_\nu(1-\gamma_5)u_\mu=O(m^{-2})\ ,\end{aligned} \tag{14a}$$

$$\begin{aligned}T_{W\phi}&\to\bar{u}_e m(1+\gamma_5)\left[\frac{\not p+\not k+m}{m^2}\right]\gamma_\nu(1-\gamma_5)u_\mu\\&=\bar{u}_e\gamma_\nu(1-\gamma_5)u_\mu=\text{const}\ ,\end{aligned} \tag{14b}$$

$$\begin{aligned}T_{\phi W}&\to\bar{u}_e\gamma_\lambda(1-\gamma_5)\left[\frac{\not p+\not k+m}{m^2}\right]m(1-\gamma_5)u_\mu\\&=\bar{u}_e\gamma_\lambda(1-\gamma_5)u_\mu=\text{const}\ ,\end{aligned} \tag{14c}$$

$$\begin{aligned}T_{\phi\phi}&\to\bar{u}_e m(1+\gamma_5)\left[\frac{\not p+\not k+m}{m^2}\right]m(1-\gamma_5)u_\mu\\&=\bar{u}_e(\not p+\not k)(1-\gamma_5)u_\mu=\text{const}\ .\end{aligned} \tag{14d}$$

Note that, in the amplitudes with only one unphysical Higgs line $T_{W\phi}$, $T_{\phi W}$, even though the power counting indicates that there are terms which are finite as $m\to\infty$, these terms do not have the right Lorentz structure to contribute to the $\mu\to e\gamma$ amplitude.

Let us apply the exact result of Eq. (11) with its limits:

$$T(0)=\tfrac{5}{6}\quad\text{and}\quad T(\infty)=\tfrac{1}{3} \tag{15}$$

to the two models mentioned in Sec. I.

### A. The sequential model

We make the simple observation that, when we apply the small-$x$ result of Eq. (8) and sum over the light neutrinos in the SM with three generations,

$$T=\sum_{1}^{3} U_{\mu i}^{*} U_{ei}\left(\tfrac{5}{6}-\tfrac{1}{4}x_i+\cdots\right) . \tag{16}$$

The unitarity of the mixing matrix then leads to the cancellation of the leading constant term and to an amplitude being suppressed by neutrino mass difference. This is just the leptonic version of the Glashow-Iliopoulos-Maiani (GIM) mechanism [13].

When we add the fourth doublet with a heavy neutrino $m_4 \gg M_W$ the amplitude becomes

$$T=\tfrac{5}{6}\sum_{i=1}^{3} U_{\mu i}^{*} U_{ei}+\tfrac{1}{3}U_{\mu 4}^{*}U_{e4}=-\tfrac{1}{2}U_{\mu 4}^{*}U_{e4} , \tag{17}$$

where we have used the unitarity property of the mixing matrix. This yields a branching ratio of

$$B(\mu\rightarrow e\gamma)=\frac{3\alpha}{8\pi}|U_{\mu 4}^{*}U_{e4}|^2 , \tag{18}$$

thus clearly showing an absence of GIM suppression as well as the evasion of the decoupling theorem. The present experimental limit then places stringent bounds on the mixing angle factors $U_{\mu 4}^{*}U_{e4}$.

### B. The seesaw model

Exactly the same results apply to the singlet heavy neutrino case. Using the angles of Eq. (2a), one has

$$B(\mu\rightarrow e\gamma)=\frac{3\alpha}{8\pi}|s^2{}_{\alpha}s_{\beta}c_{\beta}c_{\beta}s_{\gamma}|^2 . \tag{19}$$

But in this case the decoupling is recovered when one takes account of the fact in this model the mixing angle $\sin\alpha$ is restricted as in Eq. (2b); hence, the process is in fact suppressed by powers of the heavy-neutrino mass. We can easily check that the exact degree of suppression (14a) is recovered as well. In other words, the small mixing angle ($\sim m^{-1}$) just cancels the large Yukawa coupling, and the $m$ dependence comes entirely from the propagator—as in the symmetric theory.

## III. $\mu\rightarrow 3e$ AND OTHER FLAVOR-CHANGING PROCESSES

Fot the decay process of $\mu\rightarrow ee\bar{e}$ there are basically three classes of diagrams (Fig. 4): (a) induced $\mu e\gamma$ vertices, (b) induced $\mu eZ$ vertices, and (c) box diagram [14]. Their invariant form factors in momentum space can be written down:

$$\Gamma_\lambda(\mu e\gamma)=\bar{u}_e(1+\gamma_5)[(\gamma_\lambda q^2-q_\lambda \not{q})f_1(x)$$
$$+im_\mu q^\nu \sigma_{\lambda\nu}f_2(x)]u_\mu , \tag{20a}$$

$$\Gamma_\lambda(\mu eZ)=f_z(x)\bar{u}_e(1+\gamma_5)\gamma_\lambda u_\mu , \tag{20b}$$

$$B(\mu 3e)=B_W(x)\bar{u}_e(1+\gamma_5)\gamma_\lambda u_\mu \bar{u}_e(1+\gamma_5)\gamma_\lambda \nu_e . \tag{20c}$$

In terms of the invariants in the effective Lagrangian (3),

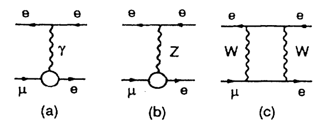



FIG. 4. Three classes of diagrams for $\mu\rightarrow 3e$ decay via photon and weak-gauge-boson exchanges.

one then finds that $f_1$ contributes to $f$, $f_2$ to $G$, $f_z$ to $E$ and $F$, and $B_W$ to $E$, etc. From (3) we can then calculate the decay rate (cf. Ref. [14]).

Similarly for other muon-number-changing processes the same nontrivial loop diagrams enter. For example, to obtain the effective Lagrangian for $\mu e$ conversion in the nucleus we have the same set of diagrams as Fig. 4 only with the $e$ lines replaced by the nonstrange quarks: $u$ and $d$. Thus again we need to concentrate on the invariant amplitudes of $\mu e\gamma$, $\mu eZ$, and the box diagram. Such a calculation has already been performed in Ref. [8]. For our purpose of studying the question of decoupling versus nondecoupling we shall merely state the large mass limit ($x\rightarrow\infty$) of their result:

$$f_1(x)\rightarrow O(\ln x) , \tag{21a}$$

$$f_2(x)\rightarrow \text{const} , \tag{21b}$$

$$f_z(x)\rightarrow O(x) , \tag{21c}$$

$$B_W(x)\rightarrow O(\ln x) . \tag{21d}$$

For a qualitative understanding of these results (up to logarithm), we have already discussed the result (21a) and (21b) for the $\mu e\gamma$ vertex in Sec. II. From (21c) we see that the $\mu eZ$ vertex grows as $m^2$ in the large mass limit. As there is a direct coupling between the $Z$ boson and the internal line of the heavy neutrino, the $\mu eZ$ diagram with a virtual Higgs boson (Fig. 5) has two heavy fermion propagators and two Yukawa couplings each proportional to $m$. Symbolically we can represent the loop integral of Fig. 5 as

$$\int d^4l\, m\frac{\not{l}+m}{l^2-m^2}\Gamma_\lambda\frac{\not{l}+m}{l^2-m^2}m\frac{1}{l^2-M_W^2} . \tag{22a}$$

The helicity structure is such that the $m$ terms in the fermion propagators can contribute, then

$$m^2m^2\int\frac{d^4l}{(l^2-m^2\cdots)^3}\rightarrow O(m^2) \tag{22b}$$

as the integration over $l$ has important contribution coming from the $l\sim m$ region. For the box diagram result of

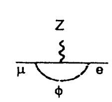



FIG. 5. Induced $\mu eZ$ vertex with intermediate states of heavy neutrino and unphysical Higgs boson.

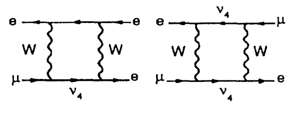



FIG. 6. Box diagrams for (a) $\mu \to 3e$, and (b) $\mu^+ e^- \to \mu^- e^+$.

(21d) we have, from Fig. 6(a),

$$m^2 \int d^4 l \frac{l^2}{(l^2 - m^2 \cdots)^4} \to \text{const} , \tag{23}$$

where the $l^2$ factor in the numerator comes from the fermion propagators, and the two powers of $m$ in front correspond to the Yukawa couplings to the heavy neutrino. In this connection, we remark that a similar box diagram [Fig. 6(b)] involving a double flavor number change (such as $\mu^+ e^- \to \mu^- e^+$ as relevant in muonium transition to antimuonium) will have an $O(m^2)$ large mass limit, as there are four powers of Yukawa couplings to the heavy neutrinos.

Let us now discuss the recovery of decoupling for the singlet neutrino case. We have already discussed the $\mu e \gamma$ case in Sec. II: its two factors of mixing angles each contributing $m^{-1}$, leading to an overall $O(m^{-2})$ suppression. Similarly for the box diagrams: each of the Yukawa couplings is matched by a mixing angle factor; this leads (up to logarithms) to an overall suppression of $m^{-2}$.

For the $\mu e Z$ vertex the same Yukawa mixing angle mechanism reduces two powers of $m$ to a constant amplitude. The final decoupling result is achieved only when one realizes that the coupling between $Z$ boson and the singlet neutrino involves an $m^{-2}$ factor itself. We can best explain this by contrasting the coupling $Z\nu_4\nu_4$ (denoted by $g_{44}$) in the sequential model (a) to that in the seesaw model (b). We can always choose to write $g_{44}$ in terms of the coupling between $Z$ and neutrino states of definite flavor (the weak eigenstates):

$$g_{44} = \sum_a U_{4a}^* U_{4a} g_a . \tag{24}$$

$g_a = g/(2\cos\theta_W)$ is the same for all flavors. Thus in the sequential model (a) where $a = e,\mu,\tau,L$ the unitarity condition of the mixing matrix immediately leads to the equality of $g_4 = g_a$. But in the seesaw model (b) there is no fourth generation and the sum over $a = e,\mu,\tau$ only leads, see Eq. (2a), to

$$g_{44} = \sin^2\alpha g_a = O(x^{-1}) g_a . \tag{25a}$$

Namely we can regard this extra suppression factor as resulting from the nonunitarity of the mixing angle factors:

$$\sum_a U_{4a}^* U_{4a} = 1 - U_{40}^* U_{40} = \sin^2\alpha . \tag{25b}$$

Taking this into account, we recover the suppression power of the symmetric theory.

We should also note that there is a subleading order-1 term in the induced $\mu e Z$ vertex coming from diagrams with nondiagonal $Z\nu_i\nu_4$ couplings (i.e., $g_{i4}$ with $i \neq 4$). This can be verified by a power-counting procedure entirely similar to that used in Eq. (22). However, for the seesaw singlet neutrino model (b) they also contribute $O(m^{-2})$ in the large mass limit, because

$$g_{i4} = \sum_a U_{ia}^* U_{4a} g_a = -U_{iL}^* U_{4L} g_a = O(m^{-1}) g_a , \tag{25c}$$

and because $U_{ie}^* U_{4\mu} = O(m^{-1})$ we again recover the $m^{-2}$ decoupling suppression.

## IV. THE $\rho$ PARAMETER

The $\rho$ parameter relates the masses of the $W$ and the $Z$ gauge bosons:

$$\rho = \frac{M_W^2}{M_z^2 \cos^2\theta_W} . \tag{26}$$

In SM $\rho$ is fixed to be one at the tree level. This comes about because after spontaneous symmetry breaking the custodian SU(2) of the SM constraints the mass term for the neutral $W_3$ to be the same as $W_1$ and $W_2$ (i.e., the charged $W$ bosons). The diagonalization of the mass matrix in the space of the $W_3$ and the U(1) gauge boson then leads to $\rho = 1$. In higher orders this result is modified by the custodian SU(2)-breaking effects through their contribution to the gauge-boson self-energies. In fact the correction $\Delta\rho$ can be expressed simply as the difference of $W_3$ and $W_1$ self-energies:

$$\Delta\rho = \frac{g^2}{M_W^2} [\Pi_{11}(0) - \Pi_{33}(0)] . \tag{27}$$

The loop integral for the vacuum-polarization diagram in Fig. 7 with the two fermion propagators having unequal masses can be calculated (omitting the couplings) to yield [15]

$$\Pi(m_1, m_2) = -\frac{1}{96\pi^2} \left[ \Lambda^2 - \frac{3}{2}(m_1^2 + m_2^2) \ln \frac{\Lambda^2}{m_1 m_2} + \frac{3}{4} \left[ \frac{m_1^4 + m_2^4}{m_1^2 - m_1^2} \right] \ln \frac{m_1^2}{m_2^2} - \frac{3}{4}(m_1^2 + m_2^2) \right] , \tag{28a}$$

where $\Lambda$ is a cutoff. For equal mass, this reduces to

$$\Pi(m, m) = -\frac{1}{96\pi^2} \left[ \Lambda^2 - 3m^2 \ln \frac{\Lambda^2}{m^2} \right] . \tag{28b}$$

For the simplest case of one doublet,

$$\begin{bmatrix} t \\ b \end{bmatrix} , \tag{29}$$

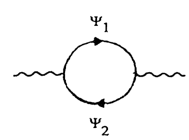



FIG. 7. The gauge-boson vacuum-polarization diagram.

we have

$$\Pi_{33}(0)=\Pi(m_t,m_t)+\Pi(m_b,m_b)\ ,$$
$$\Pi_{11}(0)=2\Pi(m_t,m_b)\ . \tag{30}$$

Equations (27) and (28) then yield [16]

$$\Delta\rho=\frac{g^2}{64\pi^2M_W^2}\left[(m_t^2+m_b^2)-\frac{2m_t^2m_b^2}{m_t^2-m_b^2}\ln\frac{m_t^2}{m_b^2}\right],$$

which for large $m_t$ leads to the well-known nondecoupling result of

$$\Delta\rho\to\frac{g^2}{64\pi^2M_W^2}m_t^2\ . \tag{31}$$

Since the large mass limit goes as $m^2$ one may wonder how the $m^{-2}$ decoupling suppression can be recovered, in view of the fact that the relevant Feynman diagram (Fig. 7) has only two vertices, thus presumably at most a suppression by two powers of the heavy-light mixing angle (each $\sim m^{-1}$). Let us first examine this question in the seesaw model (b). We shall, for simplicity of presentation, take the unessential mixing angles ($\beta$ and $\gamma$) among light neutrinos to zero (i.e., $\nu_e=\nu_1$ and $\nu_\mu=\nu_2$). Then only the tau neutrino is an admixture of the heavy component $\nu_4$ and the superlight component $\nu_3$:

$$\begin{pmatrix} c_\alpha\nu_3+s_\alpha\nu_4 \\ \tau\end{pmatrix}; \tag{32}$$

we have

$$\Pi_{33}(0)=\Pi(m_\tau,m_\tau)+c_\alpha^4\Pi(m_3,m_3)+s_\alpha^4\Pi(m_4,m_4)+2s_\alpha^2c_\alpha^2\Pi(m_3,m_4)\ ,$$
$$\Pi_{11}(0)=2c_\alpha^2\Pi(m_\tau,m_3)+2s_\alpha^2\Pi(m_\tau,m_4)\ . \tag{33}$$

These quantities have, in addition to a common divergent factor, the large-$m_4$ limits

$$\Pi_{33}(0)\to-\frac{1}{64\pi^2}(2s_\alpha^2m_4^2\ln m_4^2-s_\alpha^2c_\alpha^2m_4^2+\cdots)\ ,$$
$$\Pi_{11}(0)\to-\frac{1}{64\pi^2}(2s_\alpha^2m_4^2\ln m_4^2-s_\alpha^2m_4^2+\cdots)\ . \tag{34}$$

Consequently, the resulting correction to the $\rho$ parameter does have the required suppression factor of $m_4^{-4}$ to overcome the original nondecoupling $m_4^2$ growth:

$$\Delta\rho\to\frac{g^2}{64\pi^2M_W^2}m_4^2s_\alpha^4\ . \tag{35}$$

It is actually not difficult to see where the $\sin^4\alpha$ factor comes from. We note from Eq. (28) that the quadratic $m_4^2$ term appears in $\Pi(m,m')$ *only* when $m\neq m'$. Thus only the vacuum-polarization diagrams involving light-heavy mixings need to be included in this consideration of $\Delta\rho$. Collecting the relevant terms in (33) we have

$$\Pi_{33}(0)-\Pi_{11}(0)=2s_\alpha^2c_\alpha^2\Pi(m_3,m_4)-2s_\alpha^2\Pi(m_\tau,m_4)\ , \tag{36}$$

which immediately leads to the result in (35). Related to this, the $m_4^2\ln m_4^2$ terms of (34) get canceled in $\Delta\rho$ because such terms, like the divergent $\Lambda^2$ and $\ln\Lambda^2$ terms, appear in both $\Pi(m,m)$ and $\Pi(m,m')$. We also remark that the subleading terms $\ln m^2$ also decouple since they are multiplied by at least one power of $\sin^2\alpha$. This decoupling result is not surprising because without the mixing the singlet fermion does not couple to the $W$ to $Z$ gauge bosons.

It is also instructive to examine this nondecoupling issue in the sequential model (a). If we also take $\nu_e=\nu_1$ and $\nu_\mu=\nu_2$, then we have the third generation doublet as in (32). What differs from the seesaw model (b) is that we must add the fourth generation:

$$\begin{pmatrix} -s_\alpha\nu_3+c_\alpha\nu_4 \\ L\end{pmatrix}. \tag{37}$$

The effect of this extra doublet is that the combined contribution of (32) and (37) eliminates all the cross (i.e., unequal mass) terms in $\Pi_{33}(0)$—just another manifestation of the GIM cancellation mechanism. Consequently the $s_\alpha^2c_\alpha^2m_4^2$ term in Eq. (34) is absent and the additional contribution by (37) to $\Pi_{11}(0)$ of a factor $c_\alpha^2m_4^2$ will lead exactly to the expected single doublet result of Eq. (31).

## V. DISCUSSION

In this paper we have examined the issue of decoupling versus nondecoupling of a superheavy neutrino in the SM. We have presented a detailed calculation of the $\mu\to e\gamma$ decay amplitude in the general $R_\xi$ gauge. Our calculation shows clearly that the resultant amplitude is nonvanishing in the large mass limit because of the contribution coming from the diagrams involving unphysical Higgs bosons (i.e., the longitudinal gauge bosons). In such diagrams the usual decoupling theorem is invalidated as the Yukawa coupling grows with the fermion mass. But if the source of the large mass does not involve a large coupling constant as is the case in the seesaw model (b) then the decoupling result is recovered when the mass dependence of the heavy-light mixing angle is taken to account.

We have discussed other higher-order processes. The feature that holds in all cases is that one can discern the decoupling by simply examining (in the nonunitary gauges) the diagrams not involving any unphysical Higgs

TABLE I. Large mass $m$ behavior of the amplitudes. The two $\mu e\gamma$ amplitudes $f_1$ and $f_2$ are defined in Eq. (20); for on-shell processes only $f_2$ contributes. One of the box amplitudes is relevant for the $\mu\to 3e$ decay and the other is for muonium-antimuonium transitions.

| | Seesaw model (decoupling) | Sequential model (nondecoupling) |
|---|---|---|
| $f_1(\mu e\gamma)$ | $m^{-2}\ln m^2$ | $\ln m^2$ |
| $f_2(\mu e\gamma)$ | $m^{-2}$ | 1 |
| $\mu eZ$ | $m^{-2}$ | $m^2$ |
| Box$(\mu e-ee)$ | $m^{-2}\ln m^2$ | $\ln m^2$ |
| Box$(\mu e-e\mu)$ | $m^{-2}$ | $m^2$ |
| $\rho$ | $m^{-2}$ | $m^2$ |

bosons. In Table I we have summarized the decoupling suppression factors for each of the cases considered. Nondecoupling comes about because of the contribution by diagrams involving unphysical Higgs bosons with its large Yukawa coupling constants. But there should not be such nondecoupling when the large mass does not originate from a large coupling, as in the singlet neutrino case. We have demonstrated in this case that the decoupling suppression is recovered when one takes into account the fact that the heavy-light mixing angle itself goes as $m^{-1}$. (However, in some cases such as the $\rho$ parameter the degree of mixing angle dependence is rather subtle.) Similarly in the case of the induced $\mu e Z$ vertex one must also include the fact that the neutral-current couplings of the singlet fermion have mass suppression factors themselves.

Of course the simplest way to see that such singlet fermion contributions obey the usual decoupling theorem is to work directly with weak eigenstates instead of the mass eigenstates. However, in such an approach one has to calculate in the less familiar situation involving nondiagonal propagators. We remark that the recovery of decoupling also includes the case where the strong Yukawa coupling involves a physical Higgs boson which does not contribute to gauge-boson masses, and thus the longitudinal $W$ coupling does not grow in the large mass limit.

It should be clear that much of the results concerning superheavy neutrinos can be generalized to situations involving heavy-quarks-singlet or -nonsinglet cases. In fact we discuss, in a separate communication [17], the loop effects of a model where a heavy singlet quark mixes with the top.

## ACKNOWLEDGMENTS

The work of T. P. C. was supported by the National Science Foundation (PHY-9006776) and the University of Missouri Weldon Spring Endowment, and that of L. F. L. by the Department of Energy (DE-AC02-76ER03066).

*Address for the 1991–1992 academic year.

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