Suppression of flavor-changing neutral-current effects due to mixings with a heavy singlet fermion

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We show that the flavor-changing neutral-current couplings to the Z boson and the Higgs boson due to mixings with a heavy singlet fermion must be suppressed by two powers of heavy singlet masses. The implications for a recently suggested scheme to evade the Collider Detector at Fermilab mass bound on the top quark are discussed.

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The decoupling theorem implies that at low energies the physical effects of heavy particles are suppressed by inverse powers of heavy masses [1]. This can be understood in terms of perturbative amplitudes being damped by the exchange of heavy virtual particles. That it holds true for the tree diagrams is fairly obvious. As for loop diagrams, one can show that all effects of heavy particles are either suppressed (when heavy particles appear in convergent integrals) or absorbed into renormalizations of the light-particle couplings (when heavy particles appear in divergence graphs). On the other hand, in field theories with spontaneous symmetry breaking such decouplings can be violated if the growth of the particle mass involves the increase of some coupling constants. For example, in these theories (such as the standard model) heavy fermions often imply the presence of large Yukawa couplings. The large-mass term in the denominator of a propagator can be compensated by the large couplings in the numerator of the amplitude.

However such nondecoupling does not happen if the heavy particle is a singlet with respect to the gauge group. (Namely its large mass does not require a large coupling.) In a recent article [2] we have emphasized the role played by the mass dependence of the mixings in the restoration of decoupling for such a heavy singlet. In certain cases, notably the ρ parameter of electroweak theories, this involves some complicated cancellation. In this paper we show that the flavor-changing neutral-current (FCNC) effects induced by the mixings of such a singlet are suppressed. We shall demonstrate that the FCNC couplings of the Z boson as well as the Yukawa couplings are down by two powers of the heavy singlet masses. How it comes about again involves some subtle cancellation. We shall also comment on the implications of our result for a recently published suggestion [3] of some possible unconventional top-quark decay modes.

For definiteness let us consider the mixing effects of adding a singlet quark to a theory with n flavors of left-handed doublets. The mass matrix for the singlet quark and for quarks having the same charge as the singlet can be diagonalized yielding eigenvalues $M_0, m_1, m_2, \ldots, m_n$, with M_0 being much bigger than m_i:

$$\bar{\Psi}'_L M \Psi'_R = \bar{\Psi}_L M_{\rm diag} \Psi_R \ , \quad U\Psi_L = \Psi'_L \ , \\ V\Psi_R = \Psi'_R \ , \quad U^\dagger M V = M_{\rm diag} \ , \tag{1}$$

where the unitary matrices $U^\dagger$ and $V^\dagger$ transform weak eigenstates $\Psi'_{L,R} = (\psi_s, \psi_a, \psi_b, \ldots)_{L,R}$ to mass eigenstates $\Psi_{L,R} = (\psi_0, \psi_1, \psi_2, \ldots, \psi_n)_{L,R}$. In other words, U diagonalizes the symmetric matrix $(MM^\dagger)$, and V diagonalizes $(M^\dagger M)$.

Because of the different transformation properties of the singlet ψ_s and the doublet (ψ_a) components, the mass matrix has the following structure (in the basis where the mass matrix for the other fermions is already diagonal):

$$M = \begin{bmatrix} Q & P \\ q & p \end{bmatrix} \tag{2}$$

where p and q are, respectively, the $(n\times n)$ and $(1\times n)$ matrices with components transforming as doublets, while the $(n\times 1)$ P and the scalar Q are singlets. One would *a priori* expect P and Q to have entries much bigger than those of p and q. It then follows immediately that the combination $(MM^\dagger)$ has the following hierarchical structure among its elements: $(0,0) \gg (i,0),(0,j) \gg (i,j)$, while all components of $(M^\dagger M)$ are comparable in magnitude. This implies that, among the elements of the U and V matrices,

$$U = \begin{bmatrix} r & \epsilon \\ \delta & k \end{bmatrix} , \quad V = \begin{bmatrix} w & z \\ y & x \end{bmatrix} \tag{3}$$

those of the row and column vectors ϵ and δ should be suppressed, only of the order of m_1/M_0. This is analogous to the seesaw mechanism for massive neutrinos [4].

We will first discuss the flavor-changing couplings be-

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tween ordinary nonsinglet quarks $i,j=1,2,\ldots,n$ to the vector boson Z:

$$g_{ij}=\sum_a U_{ai}^* U_{aj} g_a = g \sum_{a\neq s} U_{ai}^* U_{aj} \quad (4)$$

where we have used the fact the Z boson couples with the same strength to each flavor of the doublets: $g_a=g$, and it does not couple to the singlet: $g_s=0$. The unitarity property of the U matrix then implies

$$g_{ij}=-g\epsilon_{is}^*\delta_{js}=O(m_i m_j/M_0^2) \quad \text{for } i\neq j \ , \quad (5)$$

showing explicitly the suppression for the flavor-changing couplings.

Next, let us examine the flavor-changing Yukawa couplings. The coupling matrix is closely related to the quark mass matrix, after a scaling by the vacuum expectation value (v) of the Higgs boson H:

$$f_y=\overline{\Psi}'_L Y \Psi'_R H \ , \quad Y=\frac{1}{v}\begin{bmatrix} 0 & 0 \\ q & p \end{bmatrix} . \quad (6)$$

Namely, the singlet entries P and Q of Eq. (2) are absent because the Higgs field H transforms as a doublet. Let us convert these couplings into those among the mass eigenstates:

$$U^\dagger Y V=\frac{1}{v}\begin{bmatrix} \delta^\dagger(qw+py) & \delta^\dagger(qz+px) \\ k^\dagger(qw+py) & k^\dagger(qz+px) \end{bmatrix} . \quad (7)$$

Here we are interested in the flavor-changing couplings corresponding to the nondiagonal elements of the $(n\times n)$ matrix $[k(qz+px)]$ and we will show that they are of the order $m_i m_j/M_0^2$.

We begin by comparing the situation to the mass matrix diagonalization in (1):

$$U^\dagger M V=\begin{bmatrix} \delta^\dagger(qw+py)+r^\dagger(Qw+Py) & \delta^\dagger(qz+px)+r^\dagger(Qz+Px) \\ k^\dagger(qw+py)+\epsilon^\dagger(Qw+Py) & k^\dagger(qz+px)+\epsilon^\dagger(Qz+Px) \end{bmatrix} . \quad (8)$$

Since the nondiagonal elements of M_{diag} must vanish, we have

$$[k^\dagger(qz+px)]_{\text{nondiag}}=-[\epsilon^\dagger(Qz+Px)]_{\text{nondiag}} \ , \quad (9)$$

$$\delta^\dagger(qz+px)=-r^\dagger(Qz+Px) \ . \quad (10)$$

Equation (10) shows that there must be a cancellation among the $O(M_0)$ elements on the right-hand side in order to match the combination of the left-hand side which is at most $O(M_0^{-1})$. Substituting (10) into (9), we have the desired flavor-changing couplings:

$$[k^\dagger(qz+px)]_{\text{nondiag}}=\frac{1}{r}[\epsilon^\dagger\delta(qz+px)]_{\text{nondiag}} \ . \quad (11)$$

The presence of the ϵ and δ factors shows that such couplings are suppressed:

$$f_{ij}=O(m_i m_j/M_0^2) \quad \text{for } i\neq j \ . \quad (12)$$

This result is easy to understand: because the singlet fermion is responsible for the existence of the flavor-changing Yukawa reactions, in the heavy mass limit, these processes must decouple together with the heavy fermion.

Let us comment on the implications of our result, Eqs. (5) and (12). Because many schemes that have been considered as possible theories beyond the standard model contain particles that are singlets with respect to the usual electroweak gauge group, there is considerable motivation to examine their possible detection, through direct or indirect means. While it is generally accepted that they are likely to be much heavier than W and Z bosons, one wonders whether their presence can affect low-energy physics through their mixings with the light-particle states. Our result will constrain such indirect effects.

For example, recently [3] Mukhopadhyaya and Nandi (MN) suggested a possibility of evading the lower mass bound for the top quark of 89 GeV established by the Collider Detector at Fermilab (CDF) Collaboration at the Tevatron. These authors postulate the existence of a singlet charge $\frac{2}{3}$ quark which mixes with the usual doublets. This allows for the presence of FCNC Yukawa couplings $\bar{t}cH$ which can dominate over a possibly suppressed $\bar{t}bW$ coupling, thus altering completely the conventional signal of the top-quark decays. A suppressed $\bar{t}bW$ coupling could come about because, with the presence of the singlet, the usual 3×3 Cabibbo-Kobayashi-Maskawa matrix, corresponding to the entry $[k]$ in (3), would no longer be unitary by itself, and the existing limits of the CKM elements could possibly allow for a suppressed $[k]_{tb}$ element (i.e., the bottom quark couples significantly to the singlet instead to the doublet t). However the discussion in this paper reminds us that the unitarity violation must be small because the extra mixing elements ϵ and δ in (3) are suppressed themselves by heavy mass factors. Furthermore, the induced FCNC Yukawa couplings are suppressed as in Eq. (12) through a rather subtle cancellation. Namely the matrix elements $f_{ij}=[k(qz+px)]_{ij}$ for $i\neq j$ is not order 1 as it appears on the surface but $O(m_i m_j M_0^{-1})$ because of the cancellation requirement of Eq. (10). These constraints limit the model of MN so much that it is not clear whether this alternative is viable at all, unless the mass of the singlet quark is taken to be artificially low.

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