Instantons and the singlet coupling in the chiral quark model

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The chiral quark model with a broken U(3) flavor symmetry can be interpreted as the effective theory of instanton-dominated nonperturbative QCD. This naturally suggests the possibility of a negative singlet to octet coupling ratio, which has been found, in a previous publication, to be compatible with the phenomenological description of the nucleon spin-flavor structure. [S0556-2821(99)06909-X]

I. INTRODUCTION

When viewed from the perspective of perturbative QCD, some of the observational data on the nucleon spin or flavor structures appear to be puzzling. The possibility that these non-trivial structures originate from nonperturbative QCD presents itself. Nonperturbative QCD naturally enters into the study of the nucleon structure because in the hadronic interior $\Lambda_{QCD}^2 \lesssim Q^2 \lesssim 1$ GeV^2, the QCD gauge coupling is expected to be large. The features in this intermediate energy regime correspond to the ''initial distributions,'' from which the observed structures at higher Q^2's are related through the standard perturbative QCD evolutions.

It has been suggested that constituent quarks and internal Goldstone bosons could be the effective degrees of freedom (DOF) for a simple description of the phenomena in this nonperturbative region [1]. Indeed, calculations at the level of the nonrelativistic chiral quark model (χQM) have been seen to yield a reasonable account of both the spin and the flavor structure [2–4]. What is the theoretical basis for such an effective DOF description? There are several distinctive theoretical approaches all leading to similar effective descriptions. They can be differentiated by theoretical (self-consistency) considerations and by model details.

The suggestion that the instanton configurations dominate the nonperturbative physics is a particularly attractive possibility [5–7]. It yields the most detailed mechanism for a dynamic breaking of chiral symmetry, and an effective theory closely resembles the χQM. In this Brief Report we shall show that one aspect of the chiral quark description can be accounted for very naturally by the instanton approach: i.e., the phenomenological suggestion of the singlet chiral meson-quark coupling having an opposite sign from that of the octet meson-quark coupling [3].

In Sec. II, we shall recall the motivation of working with a χQM having a nonet of pseudoscalar mesons, with a negative singlet-coupling as suggested by phenomenology. In Sec. III, we shall briefly recount the instanton liquid model; how the 't Hooft determinantal interaction naturally suggests a negative singlet to octet coupling ratio.

II. CHIRAL QUARK MODEL WITH A BROKEN U(3) SYMMETRY

The chiral quark idea [8] is that the QCD coupling, as it increases when proceeding to longer distance scales, could trigger the nonperturbative phenomenon of spontaneous chiral symmetry breaking before reaching the confinement radius. Thus, in the intermediate range of $\Lambda_{QCD}^2 \lesssim Q^2 \lesssim \Lambda_{\chi sb}^2 \approx 1$ GeV^2, the effective DOF are massive constituent quarks and internal Goldstone bosons (IGBs). For a better understanding, it is important to separate out the coupling *vs* mass effects. For example, in the study of the strange quark content of the nucleon, the SU(3) symmetric pion-nucleon sigma term calculation [9,10] implies a rather large strange quark content, $\bar{s} > \bar{u}, \bar{d}$, while another deduction from the neutrino charm production (without invoking the $m_s = m_u = m_d$ approximation) suggests [11,12,4] a more moderate situation $\bar{s} \simeq (\bar{u} + \bar{d})/2$.

In a previous publication [3], we have suggested the consideration of a chiral quark model with nonet of pseudoscalar mesons (instead of just the usual octet). This was mainly motivated by the theoretical consideration that in any description of the strong interaction involving three light flavors of quarks, we would start out with nine (unmixed) degenerate pseudoscalar mesons—hence an U(3) flavor symmetry. This is the case in the leading $1/N_c$ (planar) approximation (N_c being the number of colors). In this limit, the quark couplings to the singlet meson and to the octet mesons must be equal, $f_1 = f_8$. This zeroth order approximation misses some essential physical features: there is no axial anomaly, i.e., an unbroken axial U(1) symmetry, and the quark sea is flavor-symmetric, $\bar{u} = \bar{d} = \bar{s}$ [2,3]. Thus any realistic description must involve a broken U(3) symmetry (due to the higher order nonplanar contributions). If one still wants to work in the simple SU(3) limit—so as to separate out the mass $m_s > m_{u,d}$ effect (from that of the coupling), it has been suggested that we should work with two independent couplings, $f_1 \neq f_8$. One does get a substantially better description of the experimental data with such a two-parameter fit [3]; furthermore, rather surprisingly, these two couplings are found (for a better phenomenology) to have opposite signs: $f_1 \simeq -f_8$.

Why would the coupling sign make a difference in such a simple quark model calculation? It enters because we must coherently add amplitudes for the process with different neutral Goldstone boson intermediate channels (GB^0):

$$q \to GB^0 + q \to \bar{q}' + q' + q \qquad (1)$$

because they produce the same final states. Hence the relative signs of the π^0, η and η' couplings give rise to an interference pattern in the production of the $\bar{q}' q'$ pairs. The amplitude for a neutral GB emission, $q \to GB^0 + q$, is given by

$$\left(f_8 \frac{\pi^0}{\sqrt{2}} + f_8 \frac{\eta}{\sqrt{6}} + f_1 \frac{\eta'}{\sqrt{3}} \right)(\bar{q}q) \qquad (2)$$

where we have not displayed any of the charged GB (as well as the nonflavor structure) in the coupling. From the quark contents of π^0, η and η' we can immediately work out the probability for the quark pair emission, $u \to (\bar{q}' q') + u$, by the valence u quark to be

$$(u\bar{u}):(d\bar{d}):(s\bar{s}) = \left(\frac{2+\zeta}{3} \right)^2 : \left(\frac{1-\zeta}{3} \right)^2 : \left(\frac{1-\zeta}{3} \right)^2 \qquad (3)$$

where $\zeta = f_1/f_8$. One notes, in particular, for $\zeta = -2$ the interference pattern is such that $u\bar{u}$ pair is not produced by the valence u quark, while the emission probabilities of $d\bar{d}$ and $s\bar{s}$ pairs are nonvanishing and equal in this SU(3) limit. Namely, the quark sea production always involves a change of the quark flavor: $u \to d\bar{d}u$, $s\bar{s}u$, $d \to u\bar{u}d$, $s\bar{s}d$, and $s \to u\bar{u}s$, $d\bar{d}s$, but $u \nrightarrow u\bar{u}u$, $d \nrightarrow d\bar{d}d$ and $s \nrightarrow s\bar{s}s$, etc. This is the limiting case. The actual phenomenological fits (mainly from the data showing $\bar{d} > \bar{u}$) suggest more of a value in the neighborhood of $\zeta \simeq -1$.

In the chiral quark model, when one includes the $m_s > m_{u,d}$ SU(3) breakings, the singlet η' channel is suppressed by the mass effect of $M_{\eta'} > M_{\eta,K} > M_\pi$ [4]. The final result is not particularly sensitive to the η' contribution. Thus, without the consideration at an intermediate stage of the equal mass approximation, the inclusion of the singlet GB contribution may not be entirely justified on phenomenological ground. However, the requirement of a (negative) f_1 seems to suggest that the underlying theory might be such that the flavor singlet meson is needed in the coupling scheme, even though its effect is dampened by the SU(3)-breaking mass effects. (This situation is analogous to the above-discussed issue of nucleon strange quark content: it is favored by coupling but suppressed by its large mass.) The relevant point is that all this should be part of the clues about the correct nonperturbative theory underlying the effective DOF description.

III. THE INSTANTON-INDUCED EFFECTIVE INTERACTIONS

Nonperturbative QCD being likely to be rather complicated when expressed directly in terms of the fundamental DOF of (current) quarks and gluons, it may well be useful to adopt a two-stage approach. In the first stage one attempts to identify the effective DOF in terms which the physics description is simple, intuitive and phenomenologically correct. At the second stage one then tries to work out the relation between these effective DOF and QCD quarks and gluons. From such a viewpoint the above discussed χQM is a first-stage description with (constituent) quarks and internal Goldstone bosons being the effective DOF.

It turns out there are several distinctive nonperturbative QCD approaches, all leading (at least at the nonrelativistic quark model level) to the χQM as the effective theory [1]. One way to differential the separate approaches is to find model details that can be checked by experimental measurement. Here we show that the instanton approach naturally contains the possibility of a negative singlet chiral quark coupling $f_1/f_8 \simeq -2$, at least in the SU(3) limit.

Let us first recall that the instanton configuration induces a determinantal interaction among the light quarks (the 't Hooft interaction [13]):

$$\mathcal{H}_I = g \det_{i,j} [\bar{q}_{iR} q_{jL} + \text{H.c.}] \qquad (4)$$

where the flavor indices $i,j = 1,2,3$ and $q_{jL} = \frac{1}{2}(1 - \gamma_5)q_j$, etc. In the instanton approach, the light quarks pick up masses (dynamic symmetry breaking) when propagating in the background of instanton fields—they are to be identifies with the constituent quarks. On the other hand, there are actually no independent propagating pseudoscalar DOF (i.e., no GB kinetic energy terms), the IGB are just short-hands for $q\bar{q}$ loop effects: $\bar{q}q$ pairs "propagate" by leaping among states associated with instantons.

This six-quark interaction in Eq. (4) implies that an instanton absorbs a left-handed quark of each flavor and emits a right-handed quark of each flavor, $\bar{u}_R u_L \bar{d}_R d_L \bar{s}_R s_L$. This provides a mechanism for produce a negatively polarized quark sea, and (in the equal mass limit) a flavor structure of $\bar{s} > \bar{d} > \bar{u}$ in the proton in qualitative agreement with the observed nucleon spin/flavor structure. In fact we also see that such an interaction would transform a $q\bar{q}$ into quark pairs of different flavors—this is just the $\zeta = -2$ case discussed in Sec. II.

It may be worthwhile to work out some detail, to see how such opposite signs arise from the determinantal interaction. Here we shall follow Hatsuda and Kunihiro [14] and use the mean field approximation—namely a composite boson field $\Phi_{ij} = \bar{q}_i(1 - \gamma_5)q_j$ can be approximated by its (classical) vacuum expectation value, $\Phi_{ii} \to \langle \Phi_{ii} \rangle \equiv \phi_{ii}$. In this way we obtain a set of four-quark vertices from the determinantal six-quark interaction as [15]

$$g[\det \Phi + \text{H.c.}] \overset{MFA}{\to} g \left[\text{Tr}(\phi \Phi^2) - \frac{1}{2} \text{Tr}\,\phi\,\text{Tr}(\Phi^2) \right.$$
$$- \text{Tr}(\phi \Phi)\text{Tr}(\Phi)$$
$$\left. + \frac{1}{2} \text{Tr}\,\phi (\text{Tr}\,\Phi)^2 + \text{H.c.} \right]. \qquad (5)$$

Since we will be working in the SU(3) limit with $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle \equiv v$, the expectation value matrix reduces to an identity matrix multiplied by a constant, $\phi = v \mathbb{1}$ and the above expression is simplified to

$$g \det \Phi \rightarrow \frac{vg}{2} [(\text{Tr } \Phi)^2 - \text{Tr}(\Phi^2)]$$

$$= vg \left(\Phi_0^2 - \frac{1}{2} \Phi_3^2 - \frac{1}{2} \Phi_8^2 \right) \qquad (6)$$

where $\Phi_0 = (\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}$, $\Phi_3 = (\bar{u}u - \bar{d}d)/\sqrt{2}$, $\Phi_8 = (\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}$. The above expression holds for the scalar and the pseudoscalar combinations—they differ by an overall sign. To cast this in the form of boson quark couplings, we just replace one of the boson fields by its quark pairs. In this way we see clearly that single coupling is twice [16] the negative of the octet couplings $f_1 = vg$ and $f_8 = -vg/2$, confirming our expectation.

Because the determinantal interaction in Eq. (4) is symmetric under $SU_L(3) \times SU_R(3)$ but not under $U_A(1)$ it will give a mass to the singlet would-be-Goldstone boson (the η' meson), thus solving the axial $U_A(1)$ problem [13]. We can picture this solution of the η' mass problem in more physical terms: The opposite signs mean that the determinantal interaction between a quark and an antiquark is an attraction in the octet channel and a repulsion in the singlet channel.

This attraction binds $\bar{q}q$ so strongly in the octet channel that the resultant state is massless (as dictated by the gap equation). On the other hand, the corresponding repulsion in the singlet channel will reduce the binding energy (from the usual effective four-quark interaction) so that the resultant singlet pseudoscalar meson is much less tightly bound and is massive. With the $U_A(1)$ solution viewed this way, the opposite singlet-octet coupling ratio is seen to be related to the determinantal repulsion in the singlet $q\bar{q}$ channel and thus ultimately to the resolution of the η' mass problem.

In summary, we have presented an argument suggesting that, in an instanton dominated nonperturbative QCD, the singlet meson quark coupling naturally comes out to be negative, which has been found in a previous publication [3] to be compatible with the phenomenology of the nucleon spin-flavor structure. This, in turn, lends some support to the idea of an instanton-dominated nonperturbative origin of the hadron structure. In this connection, we wish to report that another argument for this result, carried out in the context of anomalous contribution to the singlet axial vector constant g_A^0 [17], will be presented in a forthcoming paper [18].

ACKNOWLEDGMENTS

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[15] One can easily understand the reduction in Eq. (5) by recalling an identity for the 3×3 matrix M:

$$\det M = M^3 - (\text{tr } M)M^2 + \frac{1}{2}[(\text{tr } M)^2 - (\text{tr } M^2)]M.$$

The trace of this identity is just the characteristic equation of a cubic equation.

[16] When SU(3) breaking in the vacuum expectation values $\langle \bar{u}u \rangle \neq \langle \bar{d}d \rangle \neq \langle \bar{s}s \rangle$ is taken into account, the magnitude of the ratio $|f_1/f_8|$ may well be reduced (from 2) for a better phenomenological fit.

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