

Introduction and overview

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- Relativity means that physically it is impossible to detect absolute motion. This can be stated as a symmetry in physics: physics equations are unchanged under coordinate transformations (i.e. when viewed by different observers).
- Special relativity (SR) is the symmetry with respect to coordinate transformations among inertial frames, general relativity (GR) among more general frames, including accelerating coordinate systems.
- The equivalence between the physics due to acceleration and to gravity means that GR is also the relativistic theory of gravitation, and SR is valid only in the absence of gravity.
- Einstein's motivations to develop GR are reviewed, and his basic idea of curved spacetime as the gravitation field is outlined.
- Relativity represents a new understanding of space and time. In SR we first learn that time is also a frame-dependent coordinate; the arena for physical phenomena is four-dimensional spacetime. GR interprets gravity as the structure of this spacetime. Ultimately, according to Einstein, space and time have no independent existence: they express the relational and causal structure of physical processes in the world.
- GR provides the natural conceptual framework for cosmology. The expanding universe reflects a dynamical spacetime. Basic features of an "exploding space" during the big bang (inflation) and the accelerated expansion during the current epoch (dark energy) can be accommodated simply by a vacuum energy term in the GR field equation, which gives rise to a gravitational repulsive force.
- The experimental foundation of GR will be emphasized in our presentation. The necessary mathematics is introduced as they are needed. After the preliminaries of Part I, we discuss the description of spacetime by the metric function in Part II. From this we can discuss many GR applications, including the study of cosmology, given in Part III. Only in Part IV do we introduce the full tensor formulation of the GR field equations and the ways to solve them.

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Einstein's general theory of relativity is a classical field theory of gravitation. It encompasses, and goes beyond, Newton's theory, which is valid only for

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particles moving with slow velocity (compared to the speed of light) in a weak and static gravitational field. Although the effects of general relativity (GR) are often small in the terrestrial and solar domains, its predictions have been accurately verified whenever high precision observations can be performed. When it comes to situations involving strong gravity, such as compact stellar objects and cosmology, the use of GR is indispensable. Einstein's theory predicted the existence of black holes, where the gravity is so strong that even light cannot escape from them. GR, with its fundamental feature of a dynamical spacetime, offers a natural conceptual framework for cosmology of an expanding universe. Furthermore, GR can simply accommodate the possibility of a constant "vacuum energy density" giving rise to a repulsive gravitational force. Such an agent is the key ingredient of modern cosmological theories of the big bang (the inflationary cosmology) and of the accelerating universe (having a dark energy).

Creating new theories for the phenomena that are not easily observed on earth poses great challenges. We cannot repeat the steps that led to the formulation of Maxwell's theory of electromagnetism, as there are not many experimental results one can use to deduce their theoretical content. What Einstein pioneered was the elegant approach of using physics symmetries as a guide to the new theories that would be relevant to the yet-to-be-explored realms. As we shall explain below, relativity is a coordinate symmetry. Symmetry imposes restrictions on the equations of physics. The condition that the new theory should be reduced to known physics in the appropriate limit often narrows it down further to a very few possibilities. The symmetry Einstein used for this purpose is the coordinate symmetries of relativity, and the guiding principle in the formulation of GR is the "principle of general covariance." In Section 1.1 we shall explain the meaning of a symmetry in physics, as well as present a brief historical account of the formulation of relativity as a coordinate symmetry. In Section 1.2 we discuss the motivations that led Einstein to his geometric view of gravitation that was GR.

Besides being a theory of gravitation, GR, also provides us with a new understanding of space and time. Starting with special relativity (SR), we learnt that time is not absolute. Just like spatial coordinates, it depends on the reference frame as defined by an observer. This leads to the perspective of viewing physical events as taking place in a 4D continuum, called the **spacetime**. Einstein went further in GR by showing that the geometry of this spacetime was just the phenomenon of gravitation and was thus determined by the matter and energy distribution. Ultimately, this solidifies the idea that space and time do not have an independent existence; they are nothing but mirroring the relations among physical events taking place in the world.

General relativity is a classical theory because it does not take into account quantum effects. GR being a theory of space and time means that any viable theory of quantum gravity must also offer a quantum description of space and time. Although quantum gravity¹ is beyond the scope of this book, we should nevertheless mention that current research shows that such a quantum theory has a rich enough structure to be a unified theory of all matter and interactions (gravitation, strong, and electroweak, etc.). Thus the quantum generalization of GR should be the fundamental theory in physics.

¹Currently the most developed study of quantum gravity is string theory. For recent textbook expositions see Zwiebach (2009), Becker, Becker, and Schwarz (2007), and Kiritsis (2007).

In this introductory chapter, we shall put forward several “big motifs” in the theory of relativity without much detailed explanation. Our purpose is to provide the reader with an overview of the subject—a roadmap, so to speak. It is hoped that, proceeding through the subsequent chapters, the reader will have occasion to refer back to this introduction, to see how various themes are substantiated.

1.1 Relativity as a coordinate symmetry

We are all familiar with the experience of sitting in a train, and not able to “feel” the speed of the train when it is moving with a constant velocity, and, when observing a passing train on a nearby track, find it difficult to tell which train is actually in motion. This can be interpreted as saying that no physical measurement can detect the absolute motion of an inertial frame. Thus we have the basic concept of **relativity**, stating that only relative motion is measurable in physics.

In this example, the passenger is an observer who determines a set of coordinates (i.e. rulers and clocks). What this observer measures is the physics with respect to this coordinate frame. The expression “the physics with respect to different coordinate systems” just means “the physics as deduced by different observers.” Physics should be independent of coordinates. Such a statement proclaims a **symmetry in physics**: Physics laws remain the same (i.e. physics equations keep the same form) under some **symmetry transformation**, which changes certain conditions, for example, the coordinates. The invariance of physics laws under coordinate transformation is called **symmetry of relativity**. This coordinate symmetry can equivalently be stated as the impossibility of any physical measurement to detect a coordinate change. Namely, if the physics remains the same in all coordinates, then no experiment can reveal which coordinate system one is in, just as the passenger cannot detect the train’s constant-velocity motion.

Rotational symmetry is a familiar example of coordinate symmetry. Physics equations are unchanged when written in different coordinate systems that are related to each other by rotations. Rotational symmetry says that it does not matter whether we do an experiment facing north or facing southwest. After discounting any peculiar local conditions, we should discover the same physics laws in both directions. Equivalently, no internal physical measurement can detect the orientation of a laboratory. The orientation of a coordinate frame is not absolute.

1.1.1 From Newtonian relativity to ether

Inertial frames of reference are the coordinate systems in which, according to Newton’s first law, a particle will, if no external force acts on it, continue its state of motion with constant velocity (including the state of rest). Galileo and Newton taught us that the physics description would be the simplest when given in these coordinate systems. The first law provides us with the definition of an inertial system (also called Galilean frames of reference). Its

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²That there should be a physical explanation why distant matter defines inertial frames was first emphasized by Bishop George Berkeley in the eighteenth century, and by Ernst Mach in the nineteenth. A brief discussion of Mach's principle can be found in Box 1.1.

³We call it Newtonian relativity as the relativity principle applied to Newtonian physics. It is also known alternatively as **Galilean relativity** as the principle of relativity was first stated by Galileo in *Dialog Concerning the Two Chief World Systems* (1632), and also as it is the symmetry with respect to Galilean transformation, cf. Eq. (2.8).

⁴While emphasizing Einstein's role, we must also point out the important contribution to SR by Henri Poincaré and Hendrik Lorentz. In fact the full Lorentz transformation was originally written down by Poincaré (who named it in Lorentz's honor). Poincaré was the first one to emphasize the view of relativity as a physics symmetry. For accessible accounts of Poincaré's contribution, see Sartori (1996) and Logunov (2001).

⁵Michelson and Morley, using a Michelson interferometer, set out to measure a possible difference in light speeds along and transverse to the orbit motion of the earth around the sun. Their null result confirmed the notion that the light speed was the same in different inertial frames.

implicit message that such coordinate systems exist is its physical content. Nevertheless, the first law does not specify which are the inertial frames in the physical universe. It is an empirical fact² that these are the frames moving at constant velocities with respect to the fixed stars—distant galaxies, or, another type of distant matter, the cosmic microwave background (CMB) radiation (see Section 10.5). There are infinite sets of such frames, differing by their relative orientation, displacement, and relative motion with constant velocities. For simplicity we shall ignore the transformations of rotation and displacement of coordinate origin, and concentrate on the relation among rectilinear moving coordinates—frames related by the **boost** transformation.

Physics equations in classical mechanics are unchanged under such boost transformations. That is, no mechanical measurement can detect the moving spatial coordinates. The familiar example of not being able to feel the speed of a moving train cited at the beginning of this section is a simple illustration of this **principle of Newtonian relativity**:³ “physics laws (classical mechanics) are the same in all inertial frames of reference.” In this sense, there is no absolute rest frame in Newtonian mechanics. The situation changed when electromagnetism was included. Maxwell showed a light speed being given by the static parameters of electromagnetism. Apparently there is only one speed of light c regardless of whether the observer is moving or not. Before Einstein, just about everyone took it to mean that Maxwell's equations were valid only in the rest frame of the **ether**, the purported medium for electromagnetic wave propagation. In effect this reintroduced into physics the notion of absolute space (the ether frame).

Also, in Newtonian mechanics the notion of time is taken to be absolute, as the passage of time is perceived to be the same in all coordinates.

1.1.2 Einsteinian relativity

It is in this context that one must appreciate Einstein's revolutionary proposal: All motions are relative and there is no need for concepts such as absolute space. Maxwell's equations are valid in every inertial coordinate system.⁴ There is no ether. Light has the peculiar property of propagating with the same speed c in all (moving) coordinate systems—as confirmed by the Michelson–Morley experiment.⁵ Furthermore, the constancy of the light speed implies that, as Einstein would show, there is no absolute time.

Einstein generalized Newtonian relativity in two stages:

- **1905** Covariance of physics laws under boost transformations were generalized from Newtonian mechanics to include electromagnetism. Namely, the laws of electricity and magnetism, as well as mechanics, are unchanged under the coordinate transformations that connect different inertial frames of reference. Einstein emphasized that this generalization implied a new kinematics: not only space but also time measurements are coordinate dependent. It is called the principle of **special relativity** because we are still restricted to the special class of coordinates: the inertial frames of reference.

- **1915** The generalization is carried out further. **General relativity** is the physics symmetry allowing for more general coordinates, including accelerating frames as well. Based on the empirical observation that the effect of an accelerating frame and gravity is the same (the principle of equivalence), GR is the field theory of gravitation; SR is special because it is valid only in the absence of gravity. GR describes gravity as curved spacetime, which is flat in SR.

To recapitulate, relativity is a coordinate symmetry. It is the statement that physics laws are the same in different coordinate systems. Thus, physically it is impossible to detect absolute motion and orientation because physics laws are unchanged under coordinate transformations. For SR, these are the transformations among Galilean frames of reference (where gravity is absent); for GR, among more general frames, including accelerating coordinate systems.

1.1.3 Coordinate symmetry transformations

Relativity is the symmetry describing the covariance of physics equations (i.e. invariance of the equation form) under coordinate transformations. We need to distinguish among several classes of transformations:

- **Galilean transformation** In classical (nonrelativistic) mechanics, inertial frames are related to each other by this transformation. Thus, by Newtonian relativity, we mean that laws of Newtonian mechanics are covariant under Galilean transformations. From the modern perspective, Galilean transformations such as $t' = t$ are valid only when the relative velocity is negligibly small compared to c .
- **Lorentz transformation** As revealed by SR, the transformation rule connecting all the inertial frames, valid for all relative speed $\leq c$, is the Lorentz transformation. That is, Galilean is the low-speed approximation of the Lorentz transformation. Maxwell's equations were first discovered to possess this symmetry—they are covariant under the Lorentz transformation. It then follows that Newtonian (nonrelativistic) mechanics must be modified so that relativistic mechanics, valid for particles having arbitrary speed up to c , can also have this Lorentz symmetry.
- **General coordinate transformation** The principle that physics equations should be covariant under the general transformations that connect different coordinate frames, including accelerating frames, is GR. Such a symmetry principle is called the **principle of general covariance**. This is the basic guiding principle for the construction of the relativistic theory of gravitation.

Thus, in GR, all sorts of coordinates are allowed—there is a “democracy of coordinate systems.” All sorts of coordinate transformations can be used. But the most fruitful way of viewing the transformations in GR is that they are **local** Lorentz transformations (i.e. an independent transformation at every spacetime point), which in the low-velocity limit are Galilean transformations.

1.1.4 New kinematics and dynamics

Einstein's formulation of the relativity principle involves a sweeping change of kinematics: not only space, but also time measurements, may differ in different inertial frames. Space and time are on an equal footing as coordinates of a reference system. We can represent space and time coordinates as the four components of a (spacetime) position vector x^μ ($\mu = 0, 1, 2, 3$), with x^0 being the time component, and the transformation for coordinate differentials is now represented by a 4×4 matrix \mathbf{A} ,

$$dx^\mu \rightarrow dx'^\mu = \sum_\nu [\mathbf{A}]_\nu^\mu dx^\nu, \quad (1.1)$$

just like a rotational coordinate transformation is represented by a 3×3 matrix. The Galilean and Lorentz transformations are **linear** transformations, i.e. the transformation matrix elements do not themselves depend on the coordinates, $[\mathbf{A}] \neq [\mathbf{A}(x)]$. That the transformation matrix is a constant with respect to the coordinates means that one makes the **same** transformation at **every** coordinate point. We call this a **global transformation**. By contrast, general coordinate transformations are **nonlinear** transformations. Recall, for example, the transformation to an accelerating frame, $x \rightarrow x' = x + vt + at^2/2$, is nonlinear in the time coordinate. Here the transformations are coordinate-dependent, $[\mathbf{A}] = [\mathbf{A}(x)]$ —a different transformation for each coordinate spacetime point. We call this a **local transformation**, or a **gauge transformation**. Global symmetry leads to kinematic restrictions, while local symmetry is a dynamics principle. As we shall see, the general coordinate symmetry (general relativity) leads to a dynamical theory of gravitation.⁶

⁶Following Einstein's seminal work, physicists learned to apply the local symmetry idea also to the internal charge-space coordinates. In this way, electromagnetism as well as other fundamental interactions among elementary particles (strong and weak interactions) can all be understood as manifestation of local gauge symmetries. For respective references of gauge theory in general and GR as a gauge theory in particular, see for example Cheng and Li (1984, 2000).

1.2 GR as a gravitational field theory

The problem of noninertial frames of reference is intimately tied to the physics of gravity. In fact, the inertial frames of reference should properly be defined as the reference frames having no gravity. GR, which includes the consideration of accelerating coordinate systems, represents a new theory of gravitation.

The development of this new theory is rather unique in the history of physics: it was not prompted by any obvious failure (crisis) of Newton's theory, but resulted from the theoretical research, "pure thought," of one person—Albert Einstein. Someone puts it this way: "Einstein just stared at his own navel, and came up with general relativity."⁷

⁷The reader of course should not take this description to imply that the discovery was in any sense straightforward and logically self-evident. In fact, it took Einstein close to 10 years of difficult research, with many false detours, to arrive at his final formulation. To find the right mathematics of Riemannian geometry, he was helped by his friend and collaborator Marcel Grossmann.

1.2.1 Einstein's motivations for the general theory

If not prompted by experimental crisis, what were Einstein's motivations in his search for this new theory? From his published papers,⁸ one can infer several **interconnected** motivations (Uhlenbeck, 1968):

⁸Einstein's classical papers in English translation may be found in the collected work published by Princeton University Press (Einstein, 1989). A less complete, but more readily available, collection may be found in Einstein *et al.* (1952).

1. To have a relativistic theory of gravitation. The Newtonian theory of gravitation is not compatible with the principle of (special) relativity as

it requires the concept of an “action-at-a-distance” force, which implies instantaneous transmission of signals.

2. To have a deeper understanding of the empirically observed equality between inertial mass and gravitational mass.⁹
3. “**Space is not a thing**” Einstein phrased his conviction that physics laws should not depend on reference frames, which express the relationship among physical processes in the world and do not have independent existence.

⁹These two types of masses will be discussed in detail in Section 4.2.1.

“**Space is not a thing**” While the first two of the above-listed motivations will be discussed further in Chapter 4, here we make some comments on the third motivation. Einstein was dissatisfied with the prevailing concept of space. SR confirms the validity of the principle of special relativity: physics is the same in every Galilean frame of reference. But as soon as one attempts to describe physical phenomena from a reference frame in acceleration with respect to an inertial frame, the laws of physics change and become more complicated because of the presence of the fictitious inertial forces. This is particularly troublesome from the viewpoint of relative motion, since one could identify either frame as the accelerating frame. (The example known as Mach’s paradox is discussed in Box 1.1.) The presence of the inertial force is associated with the choice of a noninertial coordinate system. Such coordinate-dependent phenomena can be thought of as brought about by space itself. Namely, space behaves as if it is the source of the inertial forces. Newton was thus compelled to postulate the existence of **absolute space**, as the origin of these coordinate-dependent forces. The unsatisfactory feature of such an explanation is that, while absolute space is supposed to have an independent existence, yet no object can act on this entity. Being strongly influenced by the teaching of Ernst Mach, Einstein emphasized that space and time should not be like a stage upon which physical events take place, thus having an existence even in the absence of physical interactions. In Mach and Einstein’s view, space and time are nothing but expressing relationships among physical processes in the world—“space is not a thing”. Such considerations led Einstein to the belief that the laws of physics should have the same form in all reference frames. Put another way, spacetime is a fixed for pre-GR physics, and in GR it is dynamic, as determined by the matter/energy distribution.

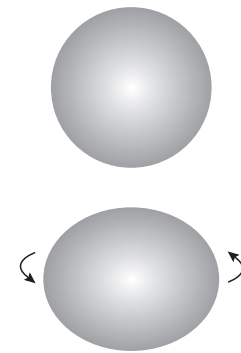


Fig. 1.1 Mach’s paradox: Two identical elastic spheres, one at rest, and the other rotating, in an inertial frame of reference. The rotating sphere is observed to bulge out in the equatorial region, taking on an ellipsoidal shape.

Box 1.1 Mach’s principle

At the beginning of his 1916 paper on general relativity, Einstein discussed **Mach’s paradox** (Fig. 1.1) to illustrate the unsatisfactory nature of Newton’s conception of space as an active agent. Consider two identical elastic spheres separated by a distance much larger than their size. One is at rest, and the other rotating around the axis joining these two spheres in an inertial frame of reference. The rotating body takes on the shape of an ellipsoid. Yet if the spheres are alone in the world, each can be regarded as being

(cont.)

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Box 1.1 (Continued)

in rotation with respect to the other. Thus there should be no reason for dissimilarity in shapes.

Mach had gone further. He insisted that it is the relative motion of the rotating sphere with respect to the distant masses that was responsible for the observed bulging of the spherical surface. The statement that the “average mass” of the universe gives rise to the inertia of an object has come to be called **Mach’s principle**. While in Einstein’s theory, the structure of space and time is influenced by the presence of matter in accordance to Mach’s idea, the question of whether GR actually incorporates all of Mach’s principle is still being debated.¹⁰ For a recent discussion see, for example, Wilczek (2004), who emphasized that even in Einstein’s theory not all coordinate systems are on an equal footing.¹¹ Thus the reader should be aware that there are subtle points with respect to the foundation questions of GR that are still topics in modern theoretical physics research.

¹⁰An affirmative answer can be argued by invoking the example of “dragging of inertial frames” by a rotating massive source, to be discussed in Section 8.4.1 (see Fig. 8.9).

¹¹This is related to the fact that the Einstein theory is a geometric theory restricted to a metric field, as discussed below.

1.2.2 Geometry as gravity

Einstein, starting with the equivalence principle (EP)—see Chapter 4—made the bold inference that the proper mathematical representation of the gravitational field is a **curved spacetime** (see Chapter 6). As a result, while spacetime has always played a passive role in our physics description, it has become a dynamic quantity in GR. Recall our experience with electromagnetism; a field theoretical description is a two-step description: the source, e.g. a proton, gives rise to a field everywhere, as described by the **field equations** (i.e. the Maxwell’s equations); the field then acts locally on the test particle, e.g. an electron, to determine its motion, as dictated by the **equation of motion** (Lorentz force law).

source \longrightarrow field \longrightarrow test particle.

GR as a field theory of gravity with curved spacetime as the gravitational field offers the same two-step description. Its essence is nicely captured in an aphorism (by John A. Wheeler):

Spacetime tells matter how to move
Matter tells spacetime how to curve.

Since a test particle’s motion in a curved space follows “the shortest possible and the straightest possible trajectory” (called the **geodesic curve**), the GR equation of motion is the **geodesic equation** (see Sections 5.2, 6.2, and 14.1). The GR field equation (the **Einstein equation**) tells us how the source of mass/energy can give rise to a curved space by fixing the curvature of the space (Sections 6.3 and 14.2). This is what we mean by saying that “GR is a geometric theory of gravity,” or “gravity is the structure of spacetime.”

1.2.3 Mathematical language of relativity

Our presentation will be such that the necessary mathematics is introduced as it is needed. Ultimately what is required for the study of GR is Riemannian geometry.

Tensor formalism Tensors are mathematical objects having definite transformation properties under coordinate transformations. The simplest examples are scalars and vector components. The principle of relativity says that physics equations should be covariant under coordinate transformation. To ensure that this principle is **automatically** satisfied, all one needs to do is to write physics equations in terms of tensors. Because each term of the equation transforms in the same way, the equation automatically keeps the same form (it is covariant) under coordinate transformations. Let us illustrate this point by the familiar example of $F_i = ma_i$ as a rotational symmetric equation. Because every term of the equation is a vector, under a rotation the same relation $F'_i = ma'_i$ holds in the new coordinate system. The physics is unchanged. We say this physics equation possesses rotational symmetry. (See Section 2.1.1 for more details.) In relativity, we shall work with tensors that have definite transformation properties under ever more general coordinate transformations: the Lorentz transformations and general coordinate transformations (see Chapters 12 and 13). If physics equations are written as tensor equations, then they are automatically relativistic. This is why a tensor formalism is needed for the study of relativity.

Our presentation will be done in the coordinate-based component formalism, although this may lack the deep geometric insight that can be provided by the coordinate-independent formulation of differential geometry. This choice is made so that the reader can study the physics of GR without overcoming the hurdle of another layer of abstraction.

Metric description vs. full tensor formulation Mathematically understanding the structure of the Einstein field equation is more difficult because it involves the Riemannian curvature tensor. A detailed discussion of the GR field equation and the ways of solving it in several simple situations will be postponed till Part IV. In Part II, our presentation will be restricted mainly to the description of space and time in the form of the metric function, which is a mathematical quantity that describes the shape of space through length measurements. From the metric function one can deduce the corresponding geodesic equation required for various applications. We will demonstrate in Part IV that the metric functions used in Parts II and III are the solutions of the Einstein field equation.

In this introductory chapter, we have emphasized the viewpoint of relativity as a coordinate symmetry. We can ensure that physics equations are covariant under coordinate transformations if they are written as tensor equations. Since the tensor formalism will not be fully explicated until Part IV, this also means that the symmetry approach will not be properly developed until later in the book, in Chapters 12–14.

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¹²QCD's gluon fields of strong interaction are examples of Yang–Mills fields. These mediating fields among quarks themselves carry, just like quarks, strong interaction charges (called “color”). Such non-Abelian gauge fields are discussed, e.g. in Cheng and Li (1984, 1988).

¹³Even the largest GR effects in terrestrial and solar domains are limited by small parameters,

$$\frac{G_{\text{N}}M_{\oplus}}{c^2R_{\oplus}} = O(10^{-10}) \text{ and}$$

$$\frac{G_{\text{N}}M_{\odot}}{c^2R_{\odot}} = O(10^{-6}),$$

with G_{N} being Newton's constant, (M_{\oplus}, R_{\oplus}) and (M_{\odot}, R_{\odot}) being respectively the (mass, radius) values of the earth and the sun.

GR as a geometric theory vs. GR as a theory of a metric field Instead of emphasizing the geometric language of general relativity, a mathematically equivalent formulation (that's even more like the field theories of other fundamental interactions) is to have GR as a theory of a metric field. The metric function is viewed as the propagating (spin-2) field of gravity, just as electromagnetic potentials is viewed as the propagating (spin-1) field of electromagnetism. This viewpoint also clarifies the origin of GR's nonlinearity. Maxwell's theory is a linear theory because the mediator of the electro magnetic (EM) interaction does not carry EM charge itself—the photon is electrically neutral. Since anything carrying energy and momentum is a source of gravity, the gravitational metric field carries energy and momentum, hence, “gravity charge” also—much in the way the Yang–Mills fields of strong and weak interactions do.¹² In Chapter 15, where we discuss gravitational waves, the metric field viewpoint of GR will be employed. In that discussion we work in the approximation of ignoring the gravity charge of the gravity waves themselves; thus, it's the linearized Einstein theory that we will be working in.

1.2.4 Observational evidence for GR

Our presentation of general relativity (GR) and cosmology will emphasize heavily the experimental foundation of these subjects. Although the effects GR are small in the terrestrial and solar domains,¹³ its predictions have been accurately verified whenever high precision observations can be performed. Notably we have the three classical tests of GR:

- the precession of the planet Mercury's perihelion, as discussed in Section 7.3.1;
- the bending of star light by the sun, in Sections 4.3.2 and 7.2.1;
- the redshift of light's frequency in a gravitational field, as in Sections 4.3.1 and 6.2.2.

An electromagnetic signal is delayed while traveling in a warped spacetime; this Shapiro time delay will be studied in Section 7.3.2. We must also use GR for situations involving time-dependent gravitational fields as in emission and propagation of gravitational waves. The existence of gravitational waves predicted by GR has been verified by observing the rate of energy loss, due to the emission of gravitational radiation, in a relativistic binary pulsar systems such as the Hulse–Taylor system (PSR B1913+16), discussed in Section 15.4.3. In recent years a most impressive set of confirmations of GR has been carried out in the newly discovered double pulsar PSR J0737 – 3039A/B (Burgay *et al.*, 2003; Lyne *et al.*, 2004).

The double pulsar system as a unique laboratory for GR tests A pulsar is a magnetized star whose rapid rotation generates a circulating plasma that serves as a source of beamed radio waves detectable on earth as periodic pulses. PSR J0737 – 3039A/B is a binary system composed of one pulsar with a period of 22 ms (pulsar A) in a 2.4-hour orbit with a younger pulsar with a period of 2.7 s (pulsar B). Neutron binaries being compact systems in rapid motion exhibit large GR effects. For example, the precession rate of

this double pulsar's periastron¹⁴ is $\dot{\omega} \simeq 17$ degrees per year,¹⁵ as compared to planet Mercury's 43 arcseconds per century. Even better, with both neutron stars being pulsars there is an abundant amount of timing data; this has allowed for more than six GR tests in one system. Each GR effect has a unique dependence on the two pulsar masses, M_A and M_B . For example the decay rate of its orbit period $\dot{P}_b(M_A, M_B)$, due to gravitational wave emission, is predicted by GR in Eq (15.71):¹⁶

$$\dot{P}_{b,GR} = -\frac{192\pi}{5c^5} \left(\frac{2\pi G_N}{P_b} \right)^{5/3} \frac{1 + (73/24)e^2 + (37/96)e^4}{(1 - e^2)^{7/2}} \frac{M_A M_B}{(M_A + M_B)^{1/3}}.$$

As a result, the measured value $\dot{P}_{b,obs} = -1.252(17) \times 10^{-12}$ can then be translated as a (double lined) curve in the “mass–mass diagram” of Fig. 1.2. The other quantities that have been measured in this system are $\dot{\omega}$, the precession of the periastron mentioned above, two parameters, r (range) and s (shape), related to the Shapiro time-delay, and the γ parameter related to special relativistic and gravitational time dilation effects. Evidently, all these curves meet at one point of $M_A = 1.33817(7)$ and $M_B = 1.2487(7)$ in units of the solar mass M_\odot . In fact, from this one can infer that GR has been verified at the impressive 0.1% level (Kramer *et al.* 2006). Furthermore, the double pulsar J0737 – 3039A/B has provided us with up-to-now the most precise test of GR's prediction of relativistic (geodetic) spin precession Ω_B with an uncertainty of only 13% (Breton *et al.*, 2008). This phenomenon can be viewed as spin–orbit coupling of the system and is (indirectly) related to the intriguing

¹⁴This is the point of closest approach between the pulsar and its companion star.

¹⁵This is more than four times the corresponding value for the Hulse–Taylor binary.

¹⁶ e is the eccentricity of the binary orbit.

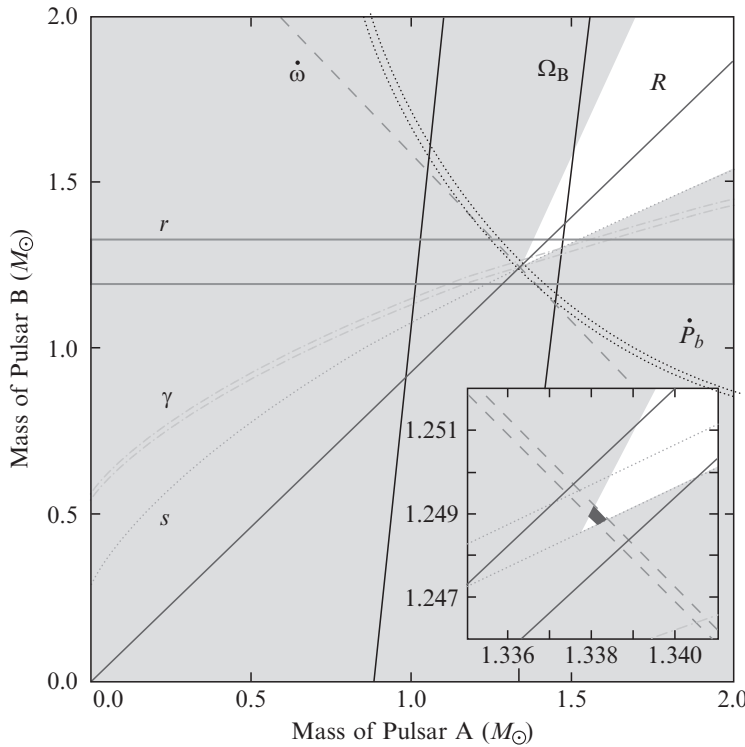


Fig. 1.2 The mass–mass plot that provides a graphical summary of GR parameters $\dot{\omega}$, γ , r , s , \dot{P}_b and Ω_B of the double pulsar J0737 – 3039A/B is from Breton *et al.* (2008); the inset is an expanded view of the region of principal interest. Since both of the projected semimajor axes $x_{A,B} = a_{A,B}(\sin i)/c$ (with i being the inclination between the binary orbit plane and the plane of the sky) have been measured, one can fix the mass ratio $R \equiv M_A/M_B = x_B/x_A$ as the two stars must be orbiting around the system's center of mass (so that $x_A M_A = x_B M_B$). The shaded area is disallowed because mass functions in this region would lead to $\sin i \geq 1$.

GR feature of “dragging of the inertial frame,” as discussed in Section 8.4 and in Problem 14.6.

1.2.5 GR as the framework for cosmology

The universe is a huge collection of matter and energy. The study of its structure and evolution, the subject of cosmology, has to be carried out in the framework of GR. The Newtonian theory for a weak and static gravitational field will not be adequate. The large collection of matter and fields means we must deal with strong gravitational effects, and to understand its evolution, the study cannot be carried out in static field theory. In fact, the very basic description of the universe is now couched in the geometric language of general relativity. A “closed universe” is one having positive spatial curvature, an “open universe” is negatively curved, etc. Thus for a proper study of cosmology, we must first learn GR.

Observationally it is clear that GR is needed to provide the conceptual framework for cosmology. The expanding universe reflects a dynamical space-time. Basic features of the big bang (inflation) and the accelerated expansion of the universe (due to dark energy) in the present epoch can be accommodated simply by a vacuum energy term (called the cosmological constant) in the GR field equation, which gives rise to the surprising feature of gravitational repulsion.

Review questions

1. What is relativity? What is the principle of special relativity? What is general relativity?
2. What is a symmetry in physics? Explain how the statement that no physical measurement can detect a particular physical feature (e.g. orientation, or the constant velocity of a lab), is a statement about a symmetry in physics. Illustrate your explanation with the examples of rotation symmetry, and the coordinate symmetry of SR.
3. In general terms, what is a tensor? Explain how a physics equation, when written in terms of tensors, automatically displays the relevant coordinate symmetry.
4. What are inertial frames of reference? Answer this in three ways.
5. The equations of Newtonian physics are unchanged when we change the coordinates from one to another inertial frame. What is this coordinate transformation?
6. The equations of electrodynamics are unchanged under another set of coordinate transformations. How are these two sets of transformations related? (You need only give their names and a qualitative description of their relation.)
7. What is the key difference between the coordinate transformations in special relativity and those in general relativity?
8. What motivated Einstein to pursue the extension of special relativity to general relativity?
9. In the general relativistic theory of gravitation, what is identified as the gravitational field? What is the GR field equation? The GR equation of motion? (Again, only the names.)
10. How does the concept of space differ in Newtonian physics and in Einsteinian (general) relativistic physics?