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# Less suppressed $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ loop amplitudes and extra dimension theories

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## Abstract

When  $\mu e\gamma$  (or  $\tau\mu\gamma$ ) loop involves a vector boson, the amplitude is suppressed by more than two powers of heavy particle masses. However we show that the scalar boson loop diagrams are much less damped. Particularly, the loop amplitude in which the intermediate fermion and scalar boson have comparable masses is as large as possible, as allowed by the decoupling theorem. Such a situation is realized in the “universal extra dimension theory”, and can yield a large enough rate for  $\mu e\gamma$  to be detectable in current experiments. Our investigation involves precise calculation of the scalar boson loop’s dependence on the masses of the intermediate states.

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## 1. Introduction

### 1.1. Probing the physics beyond the SM through the $\mu e\gamma$ loop effect

The Standard Model with massless neutrinos automatically conserves lepton flavors: the electron, muon and tau numbers. The ever stronger experimental evidence for neutrino oscillation [1] shows clearly that lepton flavor is not conserved in nature. If we accom-

modate this feature simply by an introduction of neutrino masses in the SM, other lepton flavor violating processes such as  $\mu \rightarrow e\gamma$  would still have so small a rate (a branching ratio  $\ll 10^{-40}$ ) that there is no hope for their detection in the foreseeable future. This is the case whether we have Dirac neutrino masses with their small values inserted by hand, or the neutrinos are Majorana particles with the smallness of their masses coming out of the seesaw mechanism. In the small Dirac mass scenario, the  $\mu e\gamma$  amplitude is suppressed by the neutrino mass difference  $\delta m_\nu^2$  over the vector boson mass  $M_W^2$ , namely, suppressed by a leptonic GIM scheme [2]. In the seesaw scenario, super-heavy singlet neutrino states are present in the usual

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left-handed flavor eigenstates. Their potentially significant contribution is nevertheless muffled by the mass-suppressed mixing angles [3]. Thus a detection of the  $\mu e \gamma$  process would signal the physics beyond the SM, beyond any neutrino mixing mechanism.

We are particularly interested in the possibility of this  $\mu e \gamma$  amplitude being less damped in theories that predict new particles around TeV [3,4]. One is curious whether the contribution by such heavy particles to the  $\mu e \gamma$  amplitude could be as large as possible while remaining compatible with the decoupling theorem [5]. For example, the suppressions mentioned in the last paragraph both involve at least two powers of heavy particle mass in the denominator. Are there situations in which the amplitude suppression is linear? In this Letter we report a precise calculation of the  $\mu e \gamma$  amplitude's dependence on its intermediate particle masses for the scalar boson loop diagrams. It shows that, when the intermediate fermion and boson masses are comparable, such a potentially detectable  $\mu e \gamma$  rate is possible. While our result is applicable in any theory that allows this lepton flavor nonconserving decay, in this Letter we present it mainly in the context of "large extra dimension theories" as their phenomenology has lately been under active discussion.

### 1.2. Large extra dimension theories

In the last few years there has been considerable interest in theories having extra dimensions, which either have their compactification scales being much larger than the Planck's length [6], or have strong curvature [7]. These theories can in principle generate the observed gauge hierarchy, for example, by having large extra-dimensional volume. Such theoretical suggestion would also provide an added impetus for the ongoing experimental effort to test Newton's gravity theory at the millimeter scale.

The hallmark of extra dimensions is the existence of Kaluza–Klein (KK) states: particles that can propagate in the compactified extra dimension have a tower of states with identical quantum numbers but ever increasing masses. For the simplest case of a scalar particle in a five-dimensional spacetime, with the extra dimension compactified into a circle (radius  $R$ ), we can expand the field  $\phi(x^\mu, x^5)$  in harmonics

$$\phi(x^\mu, x^5) = \sum_n \phi_n(x^\mu) e^{ip_5 x^5}. \quad (1)$$

Because the extra dimension is a circle, the positions  $x^5$  and  $x^5 + 2\pi R$  are identified, with  $\phi(x^\mu, x^5 + 2\pi R) = \phi(x^\mu, x^5)$  leading to the quantization of the extra dimension momentum:  $p_5 = n/R$ , where  $n = 0, 1, 2, \dots$  is the KK number. This implies a mass spectrum of  $M^2(n) = M^2(0) + n^2/R^2$ , where the zero mode bare mass  $M(0)$  is expected to be much smaller than the KK excitation energy of  $1/R$ . Thus there would be a tower of KK states associated with any particle that can propagate in an extra dimension.

### 1.3. Brane vs bulk particles: the universal extra dimension theory

Different extra dimension theories often have different particle assignments vis-a-vis whether they can propagate in the full higher-dimensional space or not. Those do are "bulk particles", and have associated KK states, while the "brane particles" are those confined (on the brane) to the four-dimensional spacetime. The original large dimension theory has all the SM particles stuck on the brane, and only graviton is a bulk field [6]. Prior investigation [8] and later variations include suggestions in which the neutrinos [9], the scalar bosons, or the vector gauge bosons [10], etc. propagate in the extra dimensions as well. Furthermore, with the presence of brane, the translational invariance in the extra-dimensional space is broken, and the corresponding extra dimension momentum (the KK number) is not conserved.

Among the different modifications of the original large extra dimension model, the most appealing suggestion, to our thinking, has been that by Appelquist, Cheng and Dobrescu [11] who proposed that *all* standard model particles, as well as graviton, can propagate in the extra dimensions, thus all particles have KK states. These are "universal extra dimensions". Because brane's presence is no longer required, translational invariance, and KK number conservation, are restored. There are no vertices involving only one non-zero KK mode. This is the key feature that allows such a large extra-dimensional theory to pass all the phenomenological tests. Appelquist et al. analyzed the current electroweak data, computed some parameters and concluded that the compactification size  $R$  could be  $1/300$  GeV for one extra dimension and  $1/400 \rightarrow 1/800$  GeV for two extra dimensions. These predictions are in the range of current or near-future

experiments. Other discussions about the experiment signatures of the universal extra dimensions can also be found in the literature [12]

#### 1.4. KK particles in the $\mu e \gamma$ loop

We are interested in estimating the  $\mu e \gamma$  rate as induced by the loop diagram in which the intermediate virtual particles (one fermion and one boson) may be KK states. We shall broadly distinguish two categories of models: in one category, theories do not have KK number conservation, and in such models it is possible that only one of the virtual particles is a heavy KK state. Here the general situation corresponds to mass limits when either the fermion mass is much larger than the boson mass, or the other way around. In the second category, we consider the KK number conserving universal extra dimension theory. Here both the fermion and boson must have the same KK number and their masses are comparable. This is so, because their mass square difference is the same as that between their zero modes, which is expected to be much smaller than the KK excitation. In order to consider the comparable mass case, we will have to compute the one loop amplitude exactly in its dependence of the intermediate particle mass ratio.

The gauge invariant decay amplitude for  $\mu(p) \rightarrow e(p-q) + \gamma(q, \epsilon)$  must have the form

$$T(\mu e \gamma) = \frac{ie}{16\pi} \epsilon_\lambda^*(q) \bar{u}_e(p-q) \sigma^{\lambda\rho} q_\rho \times [A_+(1 + \gamma_5) + A_-(1 - \gamma_5)] u_\mu(p). \quad (2)$$

This corresponds to a dimension-five Lagrangian density term  $\bar{\psi}_e \sigma^{\lambda\rho} \psi_\mu F_{\lambda\rho}$ . The invariant amplitudes  $A_\pm$  are induced by finite and calculable loop diagrams and proportional to an inverse mass power.

## 2. Vector loop amplitude

In the SM with one doublet of Higgs bosons and small Dirac neutrino masses, there is only one type of loop diagrams (Fig. 1) for the  $\mu e \gamma$  decay. They have a charged intermediate boson in the loop:  $\mu^- \rightarrow (\nu_i W_\gamma^-) \rightarrow e^-$  where the photon is emitted by the charged  $W$  boson in the loop (as denoted by the subscript  $\gamma$ ).

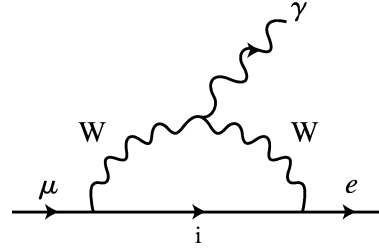


Fig. 1.  $\mu \rightarrow e \gamma$  as mediated by a vector loop. Contributions by diagrams with photon emitted by the external leptons must also be included in the calculation.

The required exact mass calculation has been performed [3,4,13] giving, in the  $m_e = 0$  approximation, the amplitudes of  $A_-^W = 0$  and

$$A_+^W = \frac{g^2 m_\mu}{8\pi M_W^2} \sum_{i=1}^3 \mathbb{U}_{\mu i}^* \mathbb{U}_{e i} F\left(\frac{m_i^2}{M_W^2}\right), \quad (3)$$

where the function<sup>2</sup>

$$F(z) = \frac{1}{6(1-z)^4} (10 - 43z + 78z^2 - 49z^3 + 18z^3 \ln z + 4z^4) \quad (4)$$

has limits

$$F(z \rightarrow \infty) \simeq \frac{2}{3} + 3 \frac{\ln z}{z}, \quad (5)$$

$$F(z \rightarrow 0) \simeq \frac{5}{3} - \frac{1}{2}z, \quad (6)$$

$$F(z \rightarrow 1) \simeq \frac{17}{12} + \frac{3}{20}(1-z). \quad (7)$$

The resultant amplitudes, after using the unitarity condition of the mixing matrices  $\sum_{i=1}^3 \mathbb{U}_{e i}^* \mathbb{U}_{\mu i} = 0$ , are summarized in Table 1.

The limit  $m_i \gg M_W$  is relevant to models in which the neutrino is a bulk field while the vector boson  $W$  is not. The amplitude  $A_+^W$  is suppressed by the heavy mass as  $(m_\mu/m_i^2) \ln m_i$ . The  $m_i \ll M_W$  case includes the specific situation of massless neutrinos  $m_i = 0$ , which leads to a vanishing amplitude, as lepton flavor must be conserved in the massless neutrino limit. For neutrinos with small (zero mode) masses  $(m_i)_0 \ll (M_W)_0$ , as well as the case when only  $W$  has KK states, the amplitude is proportional

<sup>2</sup> The sign in front of the log term was incorrectly written down in [3].

Table 1

The vector loop amplitudes $A_+^W$	
Limits	$A_+^W$
$m_i \gg M_W$	$\frac{3g^2 m_\mu}{8\pi} \sum_{i=1}^3 \left( \frac{1}{m_i^2} \ln \frac{m_i^2}{M_W^2} \right) \mathbb{U}_{\mu i}^* \mathbb{U}_{ei}$
$m_i \ll M_W$	$-\frac{g^2 m_\mu}{16\pi M_W^4} \sum_{i=1}^3 m_i^2 \mathbb{U}_{\mu i}^* \mathbb{U}_{ei}$
$m_i \simeq M_W$	$\frac{3g^2 m_\mu}{160\pi M_W^2} \sum_{i=1}^3 \frac{M_W^2 - m_i^2}{M_W^2} \mathbb{U}_{\mu i}^* \mathbb{U}_{ei}$

to neutrino mass-squared  $m_\mu m_i^2 / M_W^4$ . This results in a branching ratio so small that the decay cannot be detected experimentally in the foreseeable future. One might think that the situation of  $m_i \simeq M_W$  could offer a better chance of having a less suppressed amplitude. But this turns out not to be so. Since, for a given KK number, the mass-square-difference is given by that of the zero modes  $(M_W^2 - m_i^2)_{n \neq 0} = (M_W^2 - m_i^2)_0$ , the amplitude is again suppressed by  $m_\mu (m_i^2)_0 / (M_W^4)_n$  leading to an undetectably small branching ratio.

### 3. Scalar loop amplitude

Although the minimal SM needs only one doublet of Higgs particles, in most extensions more Higgs doublets are introduced. For example, in supersymmetry two Higgs doublets with opposite hypercharges are required. Several versions of compactification of superstring theory leads to  $E_6$  grand unified theories where each generation of leptons and quarks has a pair of oppositely hypercharged Higgs scalar boson. Thus there is strong motivation to consider theories with multiples of scalar bosons. Here we are interested how such scalars, and their possible KK excitations, can contribute to the  $\mu e \gamma$  loop amplitude [14]. Just as in the vector loop case, there is the need to obtain the exact intermediate mass dependence. We have performed this task and obtained the following result.

In Fig. 2(a) we have intermediate states of a charged scalar boson and a neutrino. Denote the Yukawa couplings of the scalar boson  $\phi$  to leptons  $l_i$  and  $l_j$  by  $y_{li}^\pm$ ,

$$\Gamma(\phi l_i l_j) = \bar{l}_i [y_{ij}^+(1 + \gamma_5) + y_{ij}^-(1 - \gamma_5)] l_j \phi + \text{h.c.} \quad (8)$$

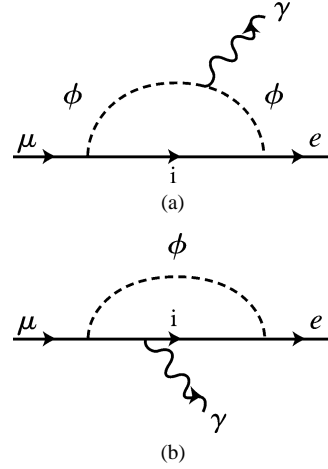


Fig. 2.  $\mu \rightarrow e \gamma$  as mediated by a loop having (a) a charged scalar, and (b) a neutral scalar, boson.

we find the amplitudes to be

$$A_+^\phi(a) = - \sum_i \frac{m_\mu}{\pi M_\phi^2} y_{ei}^+ y_{\mu i}^- G(r) - \sum_i \frac{m_i}{\pi M_\phi^2} y_{ei}^+ y_{\mu i}^+ I(r), \quad (9)$$

$$A_-^\phi(a) = - \sum_i \frac{m_\mu}{\pi M_\phi^2} y_{ei}^- y_{\mu i}^+ G(r) - \sum_i \frac{m_i}{\pi M_\phi^2} y_{ei}^- y_{\mu i}^- I(r), \quad (10)$$

where  $r \equiv \frac{m_i^2}{M_\phi^2} - 1$  and the two functions being:

$$G(r) = \frac{1}{3r} + \frac{3}{2r^2} + \frac{1}{r^3} - \frac{(1+r)^2}{r^4} \ln(1+r), \quad (11)$$

$$I(r) = \frac{1}{r} + \frac{2}{r^2} - \frac{2}{r^2} \ln(1+r) - \frac{2}{r^3} \ln(1+r). \quad (12)$$

In Fig. 2(b) we have intermediate states of a neutral scalar boson and a charged lepton. The amplitudes are

$$A_+^\phi(b) = \sum_i \frac{m_\mu}{\pi M_\phi^2} y_{ei}^+ y_{\mu i}^- H(r) + \sum_i \frac{m_i}{\pi M_\phi^2} y_{ei}^+ y_{\mu i}^+ K(r), \quad (13)$$

Table 2  
The scalar loop amplitudes  $A_+$

Limits	$A_+$	$A_+^{(\text{sum})}$
$m_i \gg M_\phi$	$\sum_i \frac{1}{\pi m_i} y_{ei}^+ y_{\mu i}^+$	$36.8 \sum_i \frac{1}{\pi m_i(1)} y_{ei}^+ y_{\mu i}^+$
$m_i \ll M_\phi$	$4 \sum_i \ln\left(\frac{M_\phi}{m_i}\right) \frac{m_i}{\pi M_\phi^2} y_{ei}^+ y_{\mu i}^+$	$\frac{2\pi}{3} \sum_i \ln\left[\frac{M_\phi(1)}{m_i}\right] \frac{m_i}{M_\phi^2(1)} y_{ei}^+ y_{\mu i}^+$
$m_i \approx M_\phi$	$\frac{1}{3} \sum_i \frac{1}{\pi M_\phi} y_{ei}^+ y_{\mu i}^+$	$12.47 \sum_i \frac{1}{\pi M_\phi(1)} y_{ei}^+ y_{\mu i}^+$

$$A_-^\phi(b) = \sum_i \frac{m_\mu}{\pi M_\phi^2} y_{ei}^- y_{\mu i}^+ H(r) + \sum_i \frac{m_i}{\pi M_\phi^2} y_{ei}^- y_{\mu i}^- K(r), \tag{14}$$

where the two functions are

$$H(r) = \frac{1}{6r} - \frac{1}{2r^2} - \frac{1}{r^3} + \frac{1+r}{r^4} \ln(1+r), \tag{15}$$

$$K(r) = \frac{1}{r} - \frac{2}{r^2} + \frac{2}{r^3} \ln(1+r). \tag{16}$$

We shall from now ignore the  $A_-$  amplitudes as they are similar to the  $A_+$  results. After making the simplifying assumption that the masses of the charged scalar boson  $M_\phi$  and neutral lepton  $m_i$  of Eq. (9) are the same as those for the neutral scalar boson and charged lepton of Eq. (13), we add the two amplitudes from (9) and (13):  $A_-^\phi(a) + A_+^\phi(b) = A_+$ . Various mass limits as shown in Eqs. (5)–(7) can be taken in a straightforward manner and we display the results<sup>3</sup> in Table 2. We have also listed, in the third column, the results when we sum over the contribution of the whole tower of KK states [15] according to the simple one-extra-dimension formula of  $M(n) = n/R = nM(1)$  when  $M(0) \simeq 0$ . Our purpose is to demonstrate that no qualitatively new feature appears in such amplitude sums, which give overall numerical coefficients and retain the same mass dependences.

The amplitude  $A_+$  in the  $m_i \simeq M_\phi$  case, as well in the  $m_i \gg M_\phi$  limit, has only one power of heavy mass in the denominator—they are as large as allowed by the decoupling theorem, which requires the amplitude to vanish when the heavy mass approaches infinity.

<sup>3</sup> Subleading terms are dropped from the results in Table 2. Also, in the  $m_i \gg M_\phi$  amplitude, the leading  $1/m_i$  terms from Fig. 2(a) and (b) actually cancel if the Yukawa couplings for the charged and neutral scalar bosons are identical. Since there is no reason to expect such an equality, we keep one of these dominant terms.

The  $m_i \ll M_\phi$  amplitude is somewhat more damped, by the heavy scalar mass  $M_\phi$  as  $m_i M_\phi^{-2} \ln M_\phi$ .

#### 4. Discussion

##### 4.1. The chiral symmetry perspective

The structure of the  $\mu e \gamma$  amplitude in Eq. (2), symbolically written as  $\bar{\psi}_L \sigma \psi_R F$  or  $\bar{\psi}_R \sigma \psi_L F$ , involves flipping the fermion chirality. Thus the amplitude must be proportional to a fermion mass. Before discussing details, we observe that if we have bulk leptons propagating in the extra dimensions, then chiral symmetry is broken in the effective four-dimensional theory by their Kaluza–Klein states, which are necessarily massive. Having a large chiral symmetry breaking, such theories offer from the outset the possibility for a less suppressed  $\mu e \gamma$  amplitude. This statement is valid whether the higher-dimensional theory has chiral symmetry or not. Our basic assumption is that the zero mode fermion masses are negligibly small compared to their KK excitation. The largest possible fermion mass that can bring about the chirality change in the  $\mu e \gamma$  amplitude is different in the vector and scalar loop diagrams.

For the vector loop contribution, we have assumed that there are only left-handed charged current couplings.<sup>4</sup> In such a situation, helicity change takes place on the external lepton lines—hence the relevant mass is that of the muon. Since the amplitude corresponds to a dimension-five operator, it must have an overall dimension of inverse-mass. Thus, in the vector loop amplitude, we expect a damping factor of  $m_\mu/M_W^2$  as

<sup>4</sup> We have not considered neutral vector loop case as such diagrams would involve further suppressions at the flavor-changing vertices.

shown in Eq. (3). If this was the principal suppression factor, the resultant amplitude and decay rate would still be large. The unitarity condition for the mixing matrices  $\sum_{i=1}^3 \mathbb{U}_{ei}^* \mathbb{U}_{\mu i} = 0$  causes the actual amplitude to be much more damped as the subleading term of the  $F$ -function in Eq. (4) generally has two additional powers of heavy masses in the denominator. Namely, a form of GIM mechanism [16] is operative here.

The scalar loop amplitudes are less suppressed for two reasons: (1) here the necessary chirality change can be effected by the large intermediate lepton mass, and (2) in the scalar case there is no cancellation mechanism  $\sum_{i=1}^3 \mathbb{U}_{ei}^* \mathbb{U}_{\mu i} = 0$ , as in the vector category, to further suppress the amplitude. For the  $m_i \gg M_\phi$  case, the heavy lepton mass  $m_i$  in the numerator flips the helicity, and its propagator provides two powers of  $m_i$  in the denominator, giving an over all  $1/m_i$  suppression. For the  $m_i \approx M_\phi$  case, either the scalar or lepton propagator can provide the mass power in the denominator. Because their masses are comparable, the resultant suppression is again  $1/m_i$ .

#### 4.2. A numerical estimate of the $\mu e \gamma$ branching ratio

We find it particularly interesting that the  $\mu e \gamma$  amplitude can be less suppressed when the intermediate scalar and lepton masses are comparable, leading to a possibly observable decay rate. From our experience with the SM, we expect the Yukawa coupling to be small, on the order of gauge coupling times the (zero mode) mass ratio of lepton over gauge boson. In particular, there has been the suggestion of neutral scalar's coupling to two charged fermions ( $i$  and  $j$ ) being on the order of  $g\sqrt{m_i m_j}/M_W$ . This coupling ansatz [17] has been studied extensively, and found to be compatible with known phenomenology. With this estimate of the Yukawa strength, the loop in Fig. 2(b) with the intermediate states being the KK states of a tau-lepton and a scalar boson would yield a branching ratio of

$$\begin{aligned} B(\mu e \gamma) &\simeq \frac{\alpha}{\pi g^4} \left( \frac{M_W^4}{m_\mu^2 M_\phi^2} \right) (y_{e\tau}^+ y_{\tau\mu}^+)^2 \\ &\simeq \frac{\alpha}{\pi} \left( \frac{m_e}{m_\mu} \right) \left( \frac{m_\tau}{M_\phi} \right)^2. \end{aligned} \quad (17)$$

For the first excited KK state with  $M_\phi = O(\text{TeV})$  and  $m_{e,\mu,\tau}$  being zero mode lepton masses, Eq. (17) gives a  $B(\mu e \gamma) = O(10^{-11})$ , which is comparable to the current experimental limit [18] of  $B(\mu e \gamma) \lesssim 1.2 \times 10^{-11}$ . This means that it is entirely conceivable that the rate predicted by the scalar loop effect is within reach of experimental detection in the near future [19].

#### 4.3. Tau decays

The considerations in this Letter can be applied directly to radiative decays of the tau lepton:  $\tau \rightarrow \mu \gamma$  and  $\tau \rightarrow e \gamma$ . To the extent that the final lepton mass being negligibly small compared to the initial lepton mass, the tau decay results involve replacing the muon mass  $m_\mu$  in equations such as Eq. (3) by the tau lepton mass  $m_\tau$ . Thus an estimate of the branching ratio for the  $\tau \mu \gamma$  decay yields

$$\begin{aligned} B(\tau \mu \gamma) &\simeq 0.17 \frac{\alpha}{\pi g^4} \left( \frac{M_W^4}{m_\tau^2 M_\phi^2} \right) (y_{\mu\tau}^+ y_{\tau\tau}^+)^2 \\ &\simeq 0.17 \frac{\alpha}{\pi} \left( \frac{m_\mu m_\tau}{M_\phi^2} \right) \end{aligned} \quad (18)$$

$$= O(10^{-10}), \quad (19)$$

where 0.17 is the branching ratio  $B(\tau \rightarrow \mu \nu \bar{\nu})$  and we have used the estimates of  $y_{\mu\tau}^+ = gm_\tau/M_W$  and  $y_{\tau\tau}^+ = g\sqrt{m_\mu m_\tau}/M_W$ . Since the existing limit [20]  $B(\tau \mu \gamma) = 1.1 \times 10^{-6}$  is still much larger than this estimate, it suggests that such a discovery would only come about after a significant increase in experimental detection efficiency has been achieved. Since our model estimate suggests  $B(\tau e \gamma) \ll B(\tau \mu \gamma)$ , it is even less likely that the decay [21]  $\tau \rightarrow e \gamma$  would be uncovered any time soon.

#### 4.4. Conclusion

We have investigated the dependence by the  $\mu e \gamma$  amplitude on heavy particle masses, finding marked difference between vector and scalar loop contributions. The vector loop amplitude, even in the comparable mass case, is strongly suppressed by powers of neutrino mass divided by the heavy mass, leading to such a small rate that the decay is predicted to be unobservable in the foreseeable future. We have calculated the precise mass-dependence of the scalar loops.

The loop amplitude with a single heavy fermion is less suppressed, not surprisingly, with one power of heavy fermion mass in the denominator. Interestingly, scalar loop amplitudes with approximately equal intermediate scalar and fermion masses (as, for example, the case in the universal extra dimension theory) are also less suppressed. Calculation using a plausible model of Yukawa couplings shows that such linearly damped amplitudes can lead to a decay rate accessible by the next generation of  $\mu e \gamma$  experiments.

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