COMMENTS ON THE NEUTRON ELECTRIC DIPOLE MOMENT IN THE WEINBERG MODEL OF *CP*-VIOLATION

T.P. CHENG
Department of Physics, University of Missouri, St. Louis, MO 63121, USA

and

Ling-Fong LI
Department of Physics, Carnegie-Mellon University, Pittsburgh, PA 15213, USA

Received 22 September 1989

The calculation of neutron EDM due to the exchange of neutral Higgs bosons is reconsidered. We find that, contrary to previous claims, both the scalar and the pseudoscalar Higgs nucleon couplings are not dominated by the heavy-quark contributions. The relevant pseudoscalar coupling is calculated by using the recent EMC data on the nucleon matrix element of the axial vector current; the scalar coupling incorporates the information contained in the πN sigma-term. The need for a proper treatment of the strange quark operators between the nucleon states is emphasized.

1. Introduction

It is well known that the neutron electric dipole moment (EDM) can play an important role in helping us to identify the sources of CP-violation [#1]. In particular, any positive result of EDM around the present experimental limit [#2] will demonstrate the presence of a *CP*-violation source above and beyond the standard Kobayashi–Maskawa mechanism. In such an eventuality, models in which the *CP*-violation is brought about by the exchange of Higgs bosons [#3] (whether involving spontaneous breaking of the *CP*-symmetry or not) are likely to receive even closer scrutiny as candidate theories of *CP*-violation. Particularly we have in mind the model by Weinberg [6], which has the attractive feature of being "*naturally flavor conserving*" and does not necessitate superheavy Higgs particles in the TeV range (hence forcing us into the superweak *CP*-violation models).

The mechanism for producing a neutron EDM that has most frequently been considered in the Weinberg model is the exchange of *neutral* Higgs bosons [7] after Anselm, Bunakov, Gudkov and Uraltsev [8] showed that previous conclusion that such contribution being suppressed by the cube of the current quark masses $m_{u,d}$ was a serious underestimate. Their analysis, and that by Shifman, Vainshtein and Zakharov [9], demonstrated, that with some seemingly reasonable assumptions, both the pseudoscalar and the scalar Higgs nucleon couplings were in fact dominated by the *heavy quark* contributions, and the resulting neutron EDM was proportional to powers of the nucleon mass. In the present paper we reconsider this problem of a proper estimate of the scalar and pseudoscalar Higgs nucleon couplings in light of the recent experimental data and improved understanding of the nucleon matrix elements of various quark bilinear operators [#4]. Our principal conclusion is that the light u, d,

[#1] For recent reviews see e.g. refs. [1,2].

[#2] The Leningrad group [3] has reported a $D_n = (14 \pm 6) \times 10^{-26}$ e-cm; more recently the Grenoble group [4] reported a $D_n = (6 \pm 4 \pm 2) \times 10^{-26}$ e-cm.

[#3] For a review see e.g. ref. [5].

[#4] The scalar coupling case has also been considered in refs. [10,11], and the pseudoscalar case in refs. [11,12]. In this paper we have gone further in applying these considerations to the neutron EDM calculation.

and s quark contributions are also important: they are in fact comparable to, or dominant over, those coming from the heavy quarks c, b and t.

In the case of the scalar coupling, the present knowledge of the chiral symmetry breaking sigma-term $\sigma_{\pi N} \simeq 60$ MeV as deduced from the πN phase-shifts and dispersion analysis [#5] actually shows that the u and d terms are comparable to the heavy quark contribution while the $m_s \bar{s}s$ term is an order of magnitude more important [10,11]. This significantly increases the scalar nucleon coupling from the value used by Anselm et al. [8]. In the case of the pseudoscalar coupling we need to reconsider ref. [8] in light of the recent result obtained by the European Muon Collaboration (EMC) in a polarized deep-inelastic scattering experiment [13]. These new data show a vanishingly small nucleon matrix element of the *singlet* axial vector current in contrast to the Anselm et al. conjecture of it being comparable to the isovector axial coupling g_A. As a result, the u, d, and s contributions are approximately equal to those by the heavy quarks. This turns out to decrease the pseudoscalar contribution. We shall also comment briefly on the issue of the momentum dependence in the form factors.

Calculating the loop diagram in fig. 1 and identifying the coefficient of the $2\bar{U}_{\mu\nu}\gamma_5 U q^\nu$ one obtains, after Wick rotation and angular integration, its contribution to the neutron EDM [8]:

[#5] For a recent summary review of issues relating to $\sigma_{\pi N}$ see e.g. ref. [10].

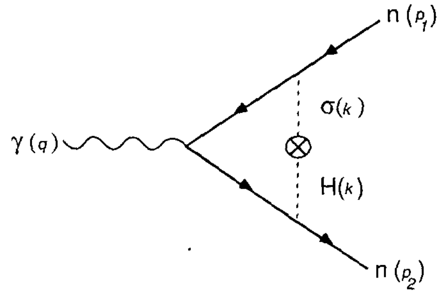

Fig. 1. Loop diagram contributing to neutron EDM. The exchanged neutral Higgs bosons are mixtures of the scalar (σ) and pseudoscalar (H) states.

$$D_n = \mu\left(\frac{f_\sigma f_H}{4\pi^2 M^2}\right)\int_0^\infty k^2\,dk^2\left(\frac{1+2M^2/k^2}{\sqrt{1+4M^2/k^2}}-1\right)$$

$$\times\langle\sigma H\rangle_k F_\sigma(k^2)F_H(k^2)\,, \qquad (1)$$

where M is the nucleon mass, and μ is the neutron magnetic moment. $f_\sigma F_\sigma(k^2)$ and $f_H F_H(k^2)$ are the scalar and pseudoscalar coupling form factors to the neutron, respectively. $\langle\sigma H\rangle_k$, the mixed propagator of the Higgs boson [#6], can be set to its value at $k^2=0$, and taken out of the integral, because we expect that the Higgs masses should be much larger than the scales of the form factors.

2. Pseudoscalar Higgs boson coupling to the neutron

To obtain the coupling f_H of the neutron to the pseudoscalar Higgs boson H we need to calculate the neutron matrix element of the quark pseudoscalar density:

$$f_H \bar{U}i\gamma_5 U = -\sum_q \frac{m_q}{v_q}\langle n|\bar{q}i\gamma_5 q|n\rangle \qquad (2)$$

where m_q and v_q are respectively the quark mass and the vacuum expectation value of the Higgs field giving rise to m_q. One then separates out the $h=c$, b, t heavy quark terms and uses "heavy quark expansion" [15,16] to relate them to the pseudoscalar density of the gluon field $G_{\mu\nu}$,

$$\bar{h}i\gamma_5 h = -\frac{\alpha_s}{16\pi m_h}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + O\left(\frac{\Lambda^3}{m_h^3}\right), \qquad (3)$$

m_h being the heavy quark mass and Λ the QCD scale factor (a few hundred MeV).

Define the neutron matrix elements [#7]

$$\xi_q \bar{U}i\gamma_5 U = \langle n|\bar{q}i\gamma_5 q|n\rangle\,,$$

$$-2M\Delta G\bar{U}i\gamma_5 U = \frac{\alpha_s}{8\pi}\langle n|G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|n\rangle\,. \qquad (4)$$

[#6] Let us recall that one of the principal uncertainties in such a calculation of the EDM is the necessary assumption about the neutral Higgs boson mixing angles. As usual we assume that they are of the same order of magnitude as those for the charged bosons which do in principle enter in the kaon CP-violation effects. For a discussion see e.g. ref. [14].

[#7] The anomaly term ΔG equals $(\alpha_s/2\pi)\Delta g$ of ref. [12].

be expressed in terms of (a different linear combination of) the matrix elements as on the RHS in eq. (14):

$$M = \langle n | m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s | n \rangle - \frac{9\alpha_s}{8\pi} \langle n | G^a_{\mu\nu} G^a_{\mu\nu} | n \rangle .$$

In this way [9] one can express the scalar coupling in terms of the light quark scalar density matrix elements only:

$$v f_\sigma \simeq -\tfrac{2}{9} M - \tfrac{7}{9} \langle n | m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s | n \rangle . \quad (15)$$

The $\bar{u}u$ and $\bar{d}d$ terms are fixed by the sigma-term [#5]

$$\sigma_{\pi N} = \tfrac{1}{2}(m_u + m_d) \langle n | \bar{u}u + \bar{d}d | n \rangle \simeq 60\, MeV . \quad (16)$$

As it turns out, $\sigma_{\pi N}$ also determines the $\bar{s}s$ term when used in conjunction with the formula for the first order SU(3) octet baryon mass shift [20]. In fact the $m_s \bar{s}s$ term is by far the largest, with a matrix element of about 350 MeV [10]. We have finally the RHS of eq. (15) to be 530 MeV,

$$v f_\sigma F_\sigma(k^2) \simeq -0.56 M \frac{1}{1 + k^2 M_0^{-2}} . \quad (17)$$

It has been suggested that this scalar channel may be dominated by the meson resonances [#11] of $C(1480)$, and $f'(1500)$. In any case we expect the form scale M_0 be around 1.5 GeV.

4. Discussion

Comparing our results with those obtained by Anselm et al. [8],

$$v f_H F_H(k^2) \simeq 2.5 \frac{1}{1 + k^2 M^{-2}} ,$$

$$v f_\sigma F_\sigma(k^2) = -0.27 M \frac{1}{1 + k^2 M^{-2}} , \quad (18)$$

we note that, after the k^2 integration in eq. (1), our change of the pseudoscalar coupling in eq. (12) *decreases* the EDM by about a factor of six, while the scalar factor of eq. (17) *increases* it by a factor about three. Consequently the net result that one obtains with our new couplings and form factors is to reduce the final result of $D_n = O(10^{-22})$ of ref. [8] by "only" a factor of two.

The principal differences between ref. [8] and this paper can be summarized as follows:

(i) For the scalar case, Anselm et al. used the heavy quark expansion to evaluate $\langle n | m_s \bar{s}s | n \rangle \simeq \frac{2}{29} M \simeq 65$ MeV, just like the rest of the heavy quarks c, b, and t.These authors assumed that the valence quark terms are negligible $\langle n | m_u \bar{u}u + m_d \bar{d}d | n \rangle \simeq 0$ presumably on ground of the smallness of the quark masses. In this paper we have shown that these assumptions may not be reliable: phenomenogical analysis of $\sigma_{\pi N}$ in fact suggests very sizable matrix elements for the light quarks $\langle n | (m_u \bar{u}u + m_d \bar{d}d) + m_s \bar{s}s | n \rangle \simeq (60 + 350)$ MeV. On the other hand, the regular heavy quark c, b, t contributions are reduced somewhat, $\langle n | m_h \bar{h}h | n \rangle \simeq 40$ MeV each, because the gluonic part of the nucleon mass is reduced [10].

(ii) For the pseudoscalar case, the coupling f_H as calculated in ref. [8] again receives negligible contributions from the u and d quark terms: Anselm et al. use the SU(2) version of the constraint equation (9):

$$\xi_u + \xi_d = 0 . \quad (19)$$

This has the consequence of turning the u, d terms of eq. (5) into an isovector combination, hence suppressed by the form factor given in eq. (11). We can see this by using eq. (19) to rewrite the original u, d pseudoscalar terms as

$$m_u \xi_u + m_d \xi_d = \tfrac{1}{2}(m_u - m_d)(\xi_u - \xi_d) = \frac{m_u - m_d}{m_u + m_d}(m_u \xi_u - m_d \xi_d) = \frac{m_u - m_d}{m_u + m_d} M (g_u - g_d) ,$$

where at the last equality we have used the $(u-d)$ axial vector divergence equation (6). Thus the RHS of this equation is proportional to the isovector $g^{(3)}$ as claimed above. The strange quark is again treated in ref. [8] as a heavy quark, and the heavy quark expansion (3) leads to $m_s \xi_s = M \Delta G$, just like the c, b, and t terms. Thus the pseudoscalar Higgs nucleon coupling is given simply by $v f_H \simeq -4 M \Delta G$. In the evaluation of the anomaly term ΔG Anselm et al. conjectured that $(g_u + g_d) \simeq (g_u - g_d)$ and this resulted in $\Delta G \simeq -0.72$. After further taking into account the form factor suppressions of the isovector part the result in eq. (18) is obtained. In our work,

[#11] See for example ref. [21].

Eq. (2) can then be expressed as

$$\nu f_H = -m_u\xi_u - m_d\xi_d - m_s\xi_s - 3M\Delta G\,, \tag{5}$$

where for simplicity the vacuum expectation value ν_q's have been taken to be equal [#8] $\nu_q=\nu$. Furthermore one can use the (anomalous) divergence equations for the axial vector current $\frac{1}{2}\bar{q}\gamma_\mu\gamma_5\lambda^{(i)}q$, taken between the neutron states:

$$Mg_q = m_q\xi_q - M\Delta G\,, \tag{6}$$

where

$$g_q\bar{U}\gamma_\mu\gamma_5 U = \langle n|\bar{q}\gamma_\mu\gamma_5 q|n\rangle\,, \tag{7}$$

to convert eq. (5) into

$$\nu f_H = -M(\sqrt{6}\,g^{(0)} + 6\Delta G)\,, \tag{8}$$

$\lambda^{(i)}$ being the Gell-Mann matrices with $\lambda^{(0)} \equiv \sqrt{\frac{2}{3}}\,\mathbb{1}$ and $g^{(0)} = (1/\sqrt{6})(g_u+g_d+g_s)$. Two points can be made about the f_H as given in eq. (8): the pseudoscalar Higgs nucleon coupling is proportional to the nucleon mass M, and (the second point) it receives an equal contribution from each quark flavor, because the recent EMC result implies that the singlet axial vector coupling $g^{(0)}$ is very small and f_H is essentially given by the last term on the RHS with one factor of $\Delta G(M/\nu)$ coming from each flavor.

Using the constraint equation expressing the fact that there is no Goldstone pole in the U(1) channel [#9],

$$\xi_u + \xi_d + \xi_s \simeq 0\,, \tag{9}$$

we can replace the anomaly term ΔG by the axial charges $g^{(i)}$ through the divergence equations (for the singlet as well as the octet currents) [12]. After using $m_s \gg m_u$, m_d we have [11]

$$\nu f_H \simeq M\left(\sqrt{6}\,g^{(0)} + 6\,\frac{m_d-m_u}{m_d+m_u}\,g(3) + 2\sqrt{3}\,g^{(8)}\right). \tag{10}$$

It turns out we can ignore not only the $g^{(0)}$ term (as explained above) but the $g^{(3)}$ contribution as well: although the isovector $g^{(3)} = \frac{1}{2}g_A = 0.63$ is sizable by itself, it is multiplied by a form factor which is suppressed by the small m_π^2 for a wide range of momenta:

$$F_H(k^2) \simeq \frac{1}{1+k^2 m_\pi^{-2}} \simeq \frac{m_\pi^2}{k^2} + \ldots\,, \tag{11}$$

because this channel is dominated by the pion pole and the form factor as defined here must be normalized to unity. Thus the most important contribution on the RHS of eq. (10) comes from the $g^{(8)}$ term, which, from SU(3) analysis, has a value [#10] of $g^{(8)} = (1/\sqrt{12})(3F-D) \simeq 0.2$. The corresponding form factor has a momentum-dependence controlled by m_η^2:

$$\nu f_H F_H(k^2) \simeq 0.7M\,\frac{1}{(1+k^2 m_\eta^{-2})}\,. \tag{12}$$

3. Scalar Higgs coupling to the neutron

For the scalar coupling [9],

$$f_\sigma \bar{U}U = -\sum_q \frac{m_q}{\nu_q}\langle n|\bar{q}q|n\rangle\,,$$

we will again treat the light and heavy quarks separately. Using the heavy quark expansion

$$\bar{h}h = -\frac{\alpha_s}{12\pi m_h}\,G^a_{\mu\nu}G^a_{\mu\nu} + \mathrm{O}\left(\frac{\Lambda^3}{m_h^3}\right) \tag{13}$$

for the c, b, t scalar densities and assuming all ν_q's to be approximately equal we get

$$\nu f_\sigma \bar{U}U = -\langle n|m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s|n\rangle + \frac{\alpha_s}{4\pi}\langle n|G^a_{\mu\nu}G^a_{\mu\nu}|n\rangle\,. \tag{14}$$

On the other hand, using the (anomalous) energy–momentum trace identity, the nucleon mass can also

[#8] The result calculated with this simplification is in effect a lower bound for the neutron EDM.

[#9] The SU(2) version of eq. (9), $\xi_u+\xi_d \simeq 0$, was first put forth by Anselm et al. [8], and another version with $\xi_u+\xi_d$ being related to the correction of the Goldberger–Treiman expression for g_A was discussed by us in ref. [12]. Because $m_s \gg m_d$, m_u, the numerical changes from using these different versions of the constraint (9) are insignificant in the EDM calculation. For a derivation of eq. (9) from chiral symmetry and $1/N$ expansion, see ref. [17]. For other applications of eq. (9) see for example refs. [18,11].

[#10] For a recent compilation $F/D \simeq 0.61$–0.64, see ref. [19].

as explained in the text following eq. (8), each flavor (including the light u, d quarks) makes an equal contribution to f_H. Furthermore, we do not treat the strange quark as a heavy flavor thus avoiding the condition $g_s = 0$, which is incompatible with the EMC experiment showing $g_s \simeq -0.23$, and obtain [12] a ΔG three to four times smaller in magnitude than the Anselm et al. value.

In this paper we wish to emphasize the need of a proper treatment of the contribution by the strange quark, which is neither very heavy nor very light. Furthermore, empirical results of $\sigma_{\pi N}$ and polarized deep inelastic scatterings (the EMC experiment) support the emerging physical picture that the strange quark content of the nucleon is much bigger than what the simple quark model would lead us to expect. Both the matrix elements $\langle n|\bar{s}s|n\rangle$ and $\langle n|\bar{s}\gamma_\mu\gamma_5 s|n\rangle$ are surprisingly large. This has opposite effects here in the calculation of the two couplings because, while the scalar density terms add, the u, d, and s axial vector pieces mutually cancel, i.e. $g^{(0)} \simeq 0$. Regardless of the final number that comes out in this EDM case, the lesson one learns is that in processes involving Higgs nucleon couplings we must treat the strange quark contributions with particular care; an improper handling of such terms can in principle make a difference of a factor of then or more.

In discussing the significance of the strange quark operator contribution, we should mention the work of Khatsymovsky, Khriplovich and Zhitnitsky [22] who also point out the relevance of this question in the calculation of neutron EDM. However, the mechanism for inducing the EDM they have concentrated on is the loop diagram involving intermediate states of K and Σ. The CP-violation is brought about by the strong operator $\bar{s}\sigma_{\mu\nu}\gamma_5 s G_{\mu\nu}$ at the $K\Sigma n$ vertex. At this stage it seems that the contribution they have examined is still not competitive when compared with the neutral Higgs exchange (excluding the question of mixing angles, see footnote 6). Nevertheless it is interesting that, within the type of "long distance effects" they were discussing, a proper treatment of the strange quark operators between the nucleon states was again an important issue.

Acknowledgement

This work is supported in part by the National Science Foundation (PHY-8907949) and in part by the Department of Energy (DE-AC02-76ER03066). One of us (T.P.C.) would like to acknowledge the hospitality of Aspen Center for Physics where part of this work was performed.

References

[1] S.M. Barr and W.J. Marciano, in: *CP* violation, ed. C. Jarlskog (World Scientific, Singapore, 1989).

[2] X.G. He, B.H.J. McKellar and S. Pakvasa, University of Hawaii report UH-511-666-89 (1989), unpublished.

[3] I.S. Altarev et al., Pis'ma Zh. Eksp. Teor. Fiz. 44 (1986) 360 [JETP Lett. 44 (1986) 460].

[4] K. Smith, in: Proc. Conf. on *CP* violation in particle physics and astrophysics (Blois, France, 1989).

[5] I.I. Bigi and A.I. Sanda, in: *CP* violation, ed. C. Jarlskog (World Scientific, Singapore, 1989).

[6] S. Weinberg, Phys. Rev. Lett. 37 (1976) 657.

[7] N.G. Deshpande and E. Ma, Phys. Rev. D 16 (1977) 1583.

[8] A.A. Anselm, V.E. Bunakov, V.P. Gudkov and N.G. Uraltsev, Phys. Lett. B 152 (1985) 116.

[9] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Phys. Lett. B 78 (1978) 443.

[10] T.P. Cheng, Phys. Rev. D 38 (1988) 2869.

[11] H.Y. Cheng, Phys. Lett. B 219 (1989) 347.

[12] T.P. cheng and L.F. Li, Phys. Rev. Lett. 62 (1989) 1441.

[13] European Muon Collab., J. Ashman et al., Phys. Lett. B 206 (1988) 364.

[14] H.Y. Cheng, Phys. Rev. D 34 (1986) 1397.

[15] A.I. Vainstein, V.I. Zakharov and M.A. Shifman, Pis'ma Zh. Eksp. Teor. Fiz. 22 (1975) 123 [JETP Lett. 22 (1975) 55.]

[16] E. Witten, Nucl. Phys. B 104 (1976) 445.

[17] S. Brodsky, J. Ellis and M. Karliner, Phys. Lett. B 206 (1988) 309.

[18] Riazuddin and Fayyazuddin, Phys. Rev. D 38 (1988) 944.

[19] M. Bourquin et al., Z. Phys. C 21 (1983) 27.

[20] T.P. Cheng, Phys. Rev. D 13 (1976) 2161.

[21] C.B. Dover and P.M. Fishbane, Phys. Rev. Lett. 62 (1989) 2917.

[22] V.M. Khatsymovsky, I.B. Khriplovich and A.R. Zhitnitsky, Z. Phys. C 36 (1987) 455.